Name: ____________________ Class: __________( ) Date: ___________________

For any updates of this book, please refer to the subject homepage:

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F5B: Chapter 7A

Date	Task	Progress	
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)
	Book Example 1	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 2	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 3	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 4	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 5	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 6	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 7	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)

	Book Example 8	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Maths Corner Exercise 7A Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 7A Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 7A Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: _________	

5B Lesson Worksheet 7.0

(Refer to Book 5B P.7.3)

Objective: To review the distance formula, equations of straight lines, nature of roots of a quadratic equation and basic properties of circles.

Distance Formula

1. Distance between $A(18, 14)$ and $B(3, 6)$

$= \sqrt{(\qquad)^2 + (\qquad)^2}$

$= \underline{\qquad}$

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. Distance between $M(5, 4)$ and $N(-7, 9)$

$=$

↪Review Ex: 1, 2

Equations of Straight Lines

3. The equation of the straight line with slope -2 and passing through $(3, 2)$ is

$y - (\quad) = (\quad)[(\quad) - (\quad)]$

$=$

$y - y_1 = m(x - x_1)$

4. The equation of the straight line with slope 5 and y-intercept -6 is

↪Review Ex: 3–8

$y = mx + b$

5. The equation of the straight line passing through $(2, 9)$ and $(0, 3)$ is

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

6. The equation of the straight line passing through $(5, -1)$ and $(1, -3)$ is

7. Consider the straight line $2x - y - 8 = 0$.

Slope $= -\dfrac{(\quad)}{(\quad)} = \underline{\qquad}$

x-intercept $= -\dfrac{(\quad)}{(\quad)} = \underline{\qquad}$

For $Ax + By + C = 0$, slope $= -\dfrac{A}{B}$, x-intercept $= -\dfrac{C}{A}$.

8. Consider the straight line $3x + 4y - 6 = 0$.

↪Review Ex: 9

Slope =

y-intercept =

y-intercept $= -\dfrac{C}{B}$

Nature of Roots of a Quadratic Equation

In each of the following, find the value of the discriminant and then find the number of real roots of the equation. **[Nos. 9–10]**

↪Review Ex: 10

9. $x^2 + 2x + 4 = 0$

$\Delta = (\quad)^2 - 4(\quad)(\quad)$

$= \underline{\qquad}$

$\Delta = b^2 - 4ac$

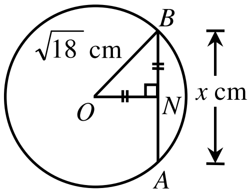
$\because$ Δ (> / = / <) 0

$\therefore$ Number of real roots $= \underline{\qquad}$

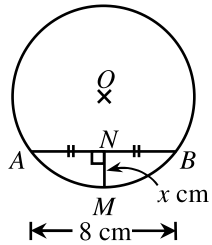
10. $5x^2 - 8x + 1 = 0$

In each of the following, O is the centre. Find the unknowns. **[Nos. 11–14]** ➥Review Ex: 12–14

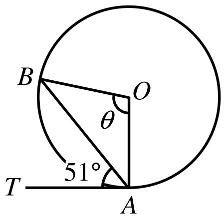
11. ANB is a straight line and $ON = BN$.



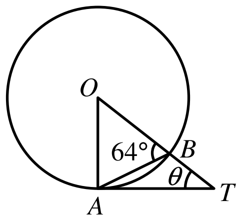
12. Radius of the circle = 5 cm
ANB is a straight line.



13. TA touches the circle at A.



14. TA touches the circle at A.
OBT is a straight line.



Level Up Question

15. The quadratic equation $x^2 + 2x + k = 3$ has two distinct real roots, where k is a positive integer. How many possible values of k are there? Explain your answer.

5B Lesson Worksheet 7.1A

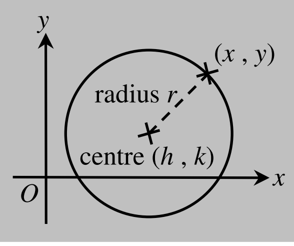
(Refer to Book 5B P.7.6)

Objective: To understand the standard form of the equations of circles.

Standard Form of the Equations of Circles

(a) The equation of a circle in the **standard form** is $(x - h)^2 + (y - k)^2 = r^2$.

(b) If the centre of a circle with radius r is at the origin (0 , 0), i.e. $h = k = 0$, the equation of the circle is $x^2 + y^2 = r^2$.



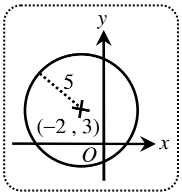
Instant Example 1

Write down the equation of the circle with centre at (−2 , 3) and radius 5 in the standard form.

The equation of the circle is

$[x - (-2)]^2 + (y - 3)^2 = 5^2$

$\underline{(x + 2)^2 + (y - 3)^2 = 25}$



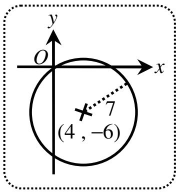
Instant Practice 1

Write down the equation of the circle with centre at (4 , −6) and radius 7 in the standard form.

The equation of the circle is

$[x - (\quad)]^2 + [y - (\quad)]^2 = (\quad)^2$

____



Write down the equation of each of the following circles in the standard form. **[Nos. 1–4]** ➥Ex 7A: 1, 2

1. A circle with centre at (0 , 0) and radius $\sqrt{8}$.

The equation of the circle is

$x^2 + (\quad)^2 = (\quad)^2$

____

r = ____

2. A circle with centre at (6 , 9) and radius 10.

h = ____
k = ____
r = ____

3. A circle with centre at (−5 , 0) and radius 2.

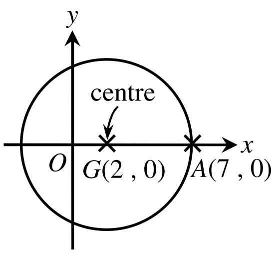
h = ____
k = ____
r = ____

4. A circle with centre at (−4 , −3) and radius $\sqrt{6}$.

h = ____
k = ____
r = ____

Instant Example 2

Find the standard equation of the circle shown in the figure.



Radius = AG

$= 7 - 2$

$= 5$

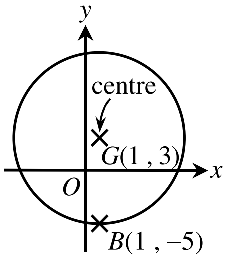
∴ The equation of the circle is

$(x - 2)^2 + (y - 0)^2 = 5^2$

$\underline{(x - 2)^2 + y^2 = 25}$

Instant Practice 2

Find the standard equation of the circle shown in the figure.



Radius = BG

$= (\quad) - (\quad)$

$= (\quad)$

∴ The equation of the circle is

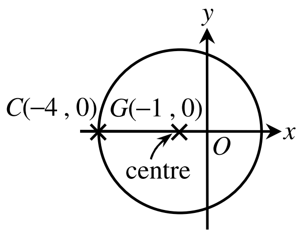
$[x - (\quad)]^2 + [y - (\quad)]^2 = (\quad)^2$

____

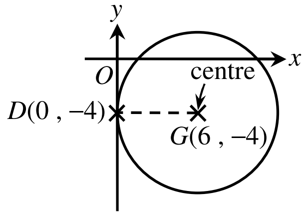
In each of the following, find the standard equation of the circle shown in the figure. **[Nos. 5–6]**

5. Radius = CG

$= (\quad) - (\quad)$



6.



➥Ex 7A: 3, 4

Instant Example 3	***Instant Practice 3***
Write down the coordinates of the centre and the radius of the circle $(x + 4)^2 + (y - 2)^2 = 49$.	Write down the coordinates of the centre and the radius of the circle $4(x - 1)^2 + 4(y + 7)^2 = 16$.
$(x + 4)^2 + (y - 2)^2 = 49$ $[x - (-4)]^2 + (y - 2)^2 = 7^2$ ← Rewrite the equation in the standard form first. ∴ <u>The coordinates of the centre are (–4 , 2) and the radius is 7.</u>	$4(x - 1)^2 + 4(y + 7)^2 = 16$ $(x - 1)^2 + (y + 7)^2 = (\quad)$ $[x - (\quad)]^2 + [y - (\quad)]^2 = (\quad)^2$ ∴ <u>The coordinates of the centre are (,) and the radius is ( ).</u>

For each of the following equations of circles, write down the coordinates of the centre and the radius of the circle. **[Nos. 7–10]**

➥Ex 7A: 5

7. $x^2 + (y - 4)^2 = 36$

$[x - (\quad)]^2 + [y - (\quad)]^2 = (\quad)^2$

8. $(x - 8)^2 + (y + 1)^2 = 25$

9. $3(x - 6)^2 + 3(y - 5)^2 = 243$

Convert the coefficients of x^2 and y^2 to 1.

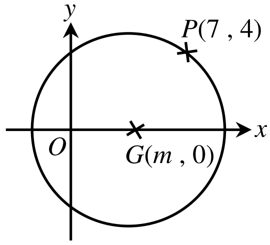
10. $5(x + 3)^2 + 5(y + 2)^2 = 15$

⇪Level Up Question⇪

11. In the figure, $G(m , 0)$ is the centre of the circle, where $0 < m < 7$. The circle passes through $P(7 , 4)$ and its radius is 5.

(a) Find the value of m.

(b) Find the equation of the circle in the standard form.



5B Lesson Worksheet 7.1B

(Refer to Book 5B P.7.9)

Objective: To understand the general form of the equations of circles.

General Form of the Equation of a Circle

(a) The equation of a circle in the **general form** is $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants.

(b) Consider a circle $x^2 + y^2 + Dx + Ey + F = 0$.

(i) Coordinates of the centre (h , k)

$= \left(-\frac{D}{2}, -\frac{E}{2}\right)$

(ii) Radius $r = \sqrt{h^2 + k^2 - F}$

$= \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$

Instant Example 1

Convert the equation of the circle $(x - 2)^2 + (y + 5)^2 = 4$ into the general form.

$(x - 2)^2 + (y + 5)^2 = 4$

$x^2 - 4x + 4 + y^2 + 10y + 25 = 4$

$\underline{x^2 + y^2 - 4x + 10y + 25 = 0}$

In the general form, make sure that the R.H.S. of the equation is 0.

Instant Practice 1

Convert the equation of the circle $(x + 3)^2 + (y - 4)^2 = 7$ into the general form.

$(x + 3)^2 + (y - 4)^2 = 7$

$x^2 + (\quad)x + (\quad) + y^2 - (\quad)y + (\quad) = 7$

____

Convert the following equations of circles into the general form. **[Nos. 1–2]** ➥Ex 7A: 6

1. $(x - 6)^2 + y^2 = 9$

$x^2 - (\quad)x + (\quad) + y^2 = 9$

In the general form,
(i) are the coefficients of x^2 and y^2 equal to 1?
(ii) is there any xy term?

2. $(x - 8)^2 + (y - 1)^2 = 12$

Instant Example 2

Find the coordinates of the centre and the radius of the circle $4x^2 + 4y^2 - 16x + 32y + 44 = 0$.

$4x^2 + 4y^2 - 16x + 32y + 44 = 0$

$x^2 + y^2 - 4x + 8y + 11 = 0$ ◄ Convert the coefficients of x^2 and y^2 to 1.

Coordinates of the centre

$= \left(-\frac{-4}{2}, -\frac{8}{2}\right)$

$= \underline{(2 , -4)}$

Radius $= \sqrt{2^2 + (-4)^2 - 11}$

$= \underline{\underline{3}}$

$D = -4$
$E = 8$
$F = 11$

Instant Practice 2

Find the coordinates of the centre and the radius of the circle $3x^2 + 3y^2 + 18x - 30y = 6$.

$3x^2 + 3y^2 + 18x - 30y = 6$

$x^2 + y^2 + (\quad)x - (\quad)y = (\quad)$

____

Coordinates of the centre

$= \left(-\frac{(\quad)}{2}, -\frac{(\quad)}{2}\right)$

$= \underline{(\quad\quad , \quad\quad)}$

Radius $= \sqrt{(\quad)^2 + (\quad)^2 - (\quad)}$

$=$ _____

$D =$ _____
$E =$ _____
$F =$ _____

Find the coordinates of the centre and the radius of each of the following circles. **[Nos. 3–6]** ➥Ex 7A: 7, 8

3. $x^2 + y^2 - 10x - 24y = 0$

$D =$ _____
$E =$ _____
$F =$ _____

Coordinates of the centre

$= \left(-\frac{(\quad)}{2}, -\frac{(\quad)}{2}\right)$

Radius =

4. $x^2 + y^2 + 8x + 12y + 27 = 0$

$D =$ _____
$E =$ _____
$F =$ _____

5. $4x - 6y - x^2 - y^2 + 3 = 0$

$D =$ _____
$E =$ _____
$F =$ _____

6. $2x^2 + 2y^2 = 34 - 32x$

$D =$ _____
$E =$ _____
$F =$ _____

7. Find the area of the circle $x^2 + y^2 + 8y + 7 = 0$ in terms of π.

$D =$ _____
$E =$ _____
$F =$ _____

8. Find the circumference of the circle $x^2 + y^2 + 2x - 14y - 14 = 0$ in terms of π.

➥Ex 7A: 15

$D =$ _____
$E =$ _____
$F =$ _____

⇪Level Up Question⇪

9. The circle $x^2 + y^2 + 4x - 2y - 20 = 0$ with centre at G passes through points P and Q.

(a) Find the coordinates of G.

Explain **(b)** Mandy claims that if $PQ = \sqrt{26}$, then $\triangle GPQ$ is an equilateral triangle. Do you agree? Explain your answer.

5B Lesson Worksheet 7.1C & D

(Refer to Book 5B P.7.12)

Objective: To determine the nature of a circle from its equation and the position of a point relative to a circle.

Nature of a Circle

Consider a circle $x^2 + y^2 + Dx + Ey + F = 0$.

(a) $\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F > 0$	(b) $\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F = 0$	(c) $\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F < 0$
A real circle	A point circle	An imaginary circle

Instant Example 1

Does each of the following equations represent a real circle, a point circle or an imaginary circle?

(a) $(x - 1)^2 + (y + 3)^2 + 15 = 0$

(b) $x^2 + y^2 - 4x - 10y + 29 = 0$

(a) $(x - 1)^2 + (y + 3)^2 + 15 = 0$

$(x - 1)^2 + (y + 3)^2 = -15$

$\because$ The R.H.S. of the equation $= -15 < 0$

$\therefore$ The equation represents an imaginary circle.

(b) $\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F = \left(\frac{-4}{2}\right)^2 + \left(\frac{-10}{2}\right)^2 - 29$

$= 0$

$\therefore$ The equation represents a point circle.

Instant Practice 1

Does each of the following equations represent a real circle, a point circle or an imaginary circle?

(a) $3(x + 9)^2 + 3(y - 2)^2 = 12$

(b) $x^2 + y^2 + 2x - 14y + 50 = 0$

(a) $3(x + 9)^2 + 3(y - 2)^2 = 12$

$(x + 9)^2 + (y - 2)^2 = (\quad)$

Convert the coefficients of x^2 and y^2 to 1.

$\because$ The R.H.S. of the equation

$= (\quad) \ (> / = / <) \ 0$

$\therefore$ The equation represents ________________.

(b) $\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F = \left(\quad\right)^2 + \left(\quad\right)^2 - (\quad)$

$= (\quad)$

$\therefore$ The equation represents ________________.

Does each of the following equations represent a real circle, a point circle or an imaginary circle? **[Nos. 1–4]**

1. $5(x - 6)^2 + 5(y - 1)^2 = 0$

$(\qquad)^2 + (\qquad)^2 = (\quad)$

2. $4(x + 3)^2 + 4(y + 7)^2 + 36 = 0$

➥Ex 7A: 9

3. $x^2 + y^2 - x + 3y + 3 = 0$

$D =$ _____

$E =$ _____

$F =$ _____

$\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F$

$=$

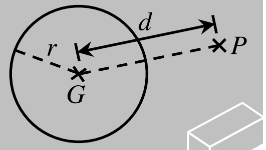
4. $2x^2 + 2y^2 + 24x + 8y + 68 = 0$

Convert the coefficients of x^2 and y^2 to 1.

Position of a Point Relative to a Circle

Suppose the distance between any point P and the centre G is d and the radius is r.

(a) P lies **outside** the circle.	**(b)** P lies **on** the circle.	**(c)** P lies **inside** the circle.
$d > r$	$d = r$	$d < r$



Instant Example 2

Determine whether $P(-3, 0)$ lies inside, outside or on the circle $(x + 3)^2 + (y - 5)^2 = 36$.

$(x + 3)^2 + (y - 5)^2 = 36$

$[x - (-3)]^2 + (y - 5)^2 = 6^2$

Coordinates of the centre = $(-3, 5)$ ← $(-3, 5)$ and P lie on the same vertical line.

Radius = 6

Distance between P and the centre = $5 - 0$

$= 5 < 6$

∴ Point P lies inside the circle.

Instant Practice 2

Determine whether $Q(3, -4)$ lies inside, outside or on the circle $(x - 1)^2 + (y + 4)^2 = 4$.

$(x - 1)^2 + (y + 4)^2 = 4$

$(\quad)^2 + [\quad]^2 = (\quad)^2$

Coordinates of the centre = (,)

Radius = ( )

Distance between Q and the centre = ( ) − ( )

= ( )

∴ Point Q lies ________________.

5. Determine whether $R(1, -5)$ lies inside, outside or on the circle $(x + 5)^2 + (y - 7)^2 = 169$.

$(x + 5)^2 + (y - 7)^2 = 169$

$[\quad]^2 + (\quad)^2 = (\quad)^2$

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

6. Determine whether $S(-6, 2)$ lies inside, outside or on the circle $(x + 8)^2 + (y + 2)^2 = 32$.

➥Ex 7A: 10–12

⇪Level Up Question⇪

7. Lily claims that circle A: $(x - 11)^2 + (y + 9)^2 = 100$ lies outside circle B: $(x - 4)^2 + (y - 15)^2 = 14^2$. Do you agree? Explain your answer.

Explain

7 Equations of Circles

Consolidation Exercise 7A

[*In this exercise, leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*]

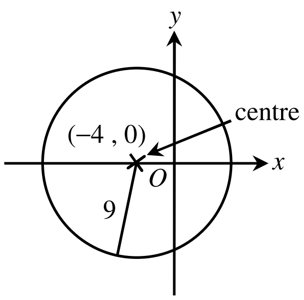
Level 1

1. Using the following centres G and radii r, write down the equations of circles in the standard form.

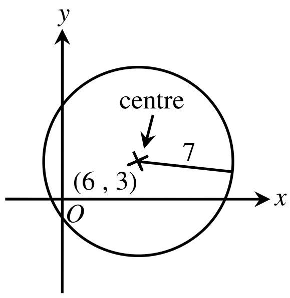
(a) Coordinates of $G = (0\ ,\ 3)$, $r = 2$
(b) Coordinates of $G = (4\ ,\ 5)$, $r = 6$
(c) Coordinates of $G = (-2\ ,\ -7)$, $r = 5$
(d) Coordinates of $G = (-1\ ,\ 4)$, $r = \sqrt{5}$
(e) Coordinates of $G = (-6\ ,\ 1)$, $r = \frac{1}{4}$
(f) Coordinates of $G = (5\ ,\ -2)$, $r = \frac{3}{2}$

2. For each of the following figures, write down the equation of the circle in the standard form.

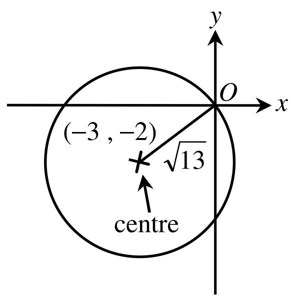
(a)



(b)

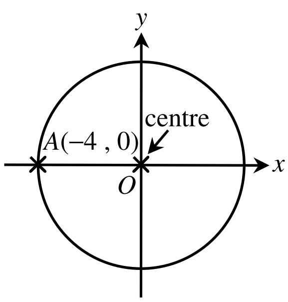


(c)

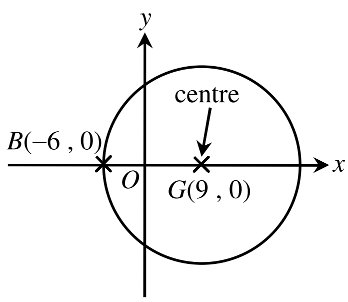


3. For each of the following, find the equation of the circle in the standard form.

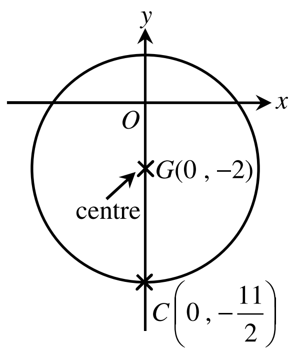
(a)



(b)

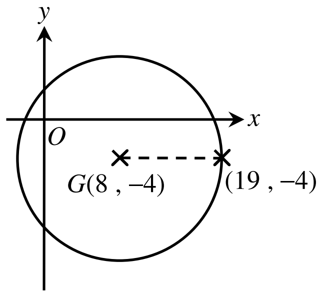


(c)

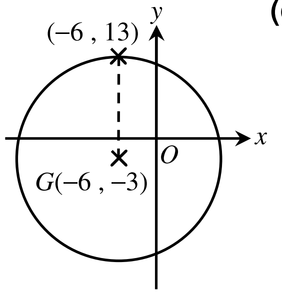


4. For each of the following, G is the centre of the circle. Write down the equation of the circle in the standard form.

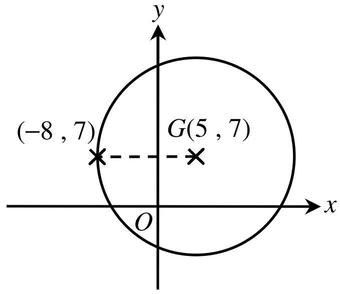
(a)



(b)



(c)



5. For each of the following equations of circles, write down the coordinates of the centre and the radius of the circle.

(a) $x^2 + y^2 = 64$ **(b)** $(x - 6)^2 + y^2 = 4$ **(c)** $(x + 1)^2 + (y - 5)^2 = 9$

(d) $(x - 3)^2 + (y + 2)^2 = 225$ **(e)** $4(x + 5)^2 + 4y^2 = 36$ **(f)** $3(x - 2)^2 + 3(y + 8)^2 = 87$

6. Convert the following equations of circles into the general form.

(a) $(x + 3)^2 + (y + 2)^2 = 25$ **(b)** $(x + 7)^2 + (y - 4)^2 = 16$

(c) $(x - 9)^2 + y^2 = 81$ **(d)** $(x - 5)^2 + (y + 5)^2 = 60$

7. For each of the following equations of circles in the general form, find the coordinates of the centre and the radius of the circle.

(a) $x^2 + y^2 - 6x + 8y = 0$ **(b)** $x^2 + y^2 - 4x - 2y + 1 = 0$

(c) $x^2 + y^2 + 12x + 8y - 29 = 0$ **(d)** $x^2 + y^2 + 10x - 2y + 5 = 0$

8. Find the coordinates of the centre and the radius of each of the following circles.

(a) $y^2 + 4x = 6y - x^2 + 3$ **(b)** $\dfrac{x^2 + y^2}{3} = 2x - 4y + 12$

(c) $7x^2 + 7y^2 - 28x = 49$ **(d)** $12y + 6x - 9 = 3x^2 + 3y^2$

9. Does each of the following equations represent a real circle, a point circle or an imaginary circle?

(a) $(x - 3)^2 + \left(y + \dfrac{2}{3}\right)^2 - 3 = 0$ **(b)** $5(x - 4)^2 + 5(y + 6)^2 = 0$

(c) $x^2 + y^2 - 4x - 8y + 21 = 0$ **(d)** $3x^2 + 3y^2 - 18x + 24y - 33 = 0$

10. Consider the circle $x^2 + y^2 + 2x + 6y - 6 = 0$.

(a) Write down the coordinates of the centre and the radius of the circle.

Explain **(b)** Is $A(0 , 1)$ a point outside the circle? Explain your answer.

11. Using the following centres G and radii r, determine whether point P lies inside, outside or on the circle.

(a) Coordinates of $G = (6 , 3)$, $r = 7$, coordinates of $P = (6 , -4)$

(b) Coordinates of $G = (-5 , 2)$, $r = 8$, coordinates of $P = (5 , 2)$

(c) Coordinates of $G = (-4 , -3)$, $r = 11$, coordinates of $P = (4 , 3)$

(d) Coordinates of $G = (1 , -2)$, $r = 4$, coordinates of $P = (-2 , 1)$

12. In each of the following, determine whether point P lies inside, outside or on the circle.

(a) Coordinates of $P = (4, 2)$, equation of the circle: $\left(x-\frac{3}{2}\right)^2 + (y+4)^2 = 36$

(b) Coordinates of $P = (-12, 3)$, equation of the circle: $x^2 + y^2 + 8x - 18y - 3 = 0$

(c) Coordinates of $P = \left(\frac{1}{2}, -1\right)$, equation of the circle: $x^2 + y^2 + 2x + 6y - 6 = 0$

13. Consider the circle $x^2 + (y+3)^2 = 16$. If $Q(0, a)$ is a point outside the circle and $a < -3$, find the range of values of a.

14. Consider the circle $(x+2)^2 + (y-3)^2 = 25$. If $Q(b, 3)$ is a point inside the circle and $b > 0$, find the range of values of b.

15. The equation of circle G is $x^2 + y^2 + 6x - 4y - 4 = 0$. The equation of straight line L is $2x + y + 4 = 0$.

(a) Write down the coordinates of the centre and the radius of the circle.

Explain **(b)** Does L pass through the centre of G? Explain your answer.

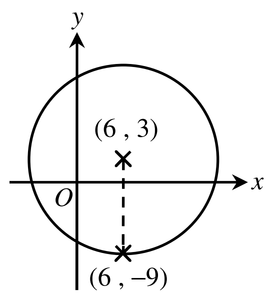
16. The equation of a circle is $2\left(x-\frac{9}{2}\right)^2 + 2(y-9)^2 = 16k^2 - 200k + 600$, where k is a constant. Find the value of k such that the equation represents a point circle.

17. Consider the circle $2x^2 + 2y^2 + 20x - 40y - 38 = 0$. Find the area and the perimeter of the circle in terms of π.

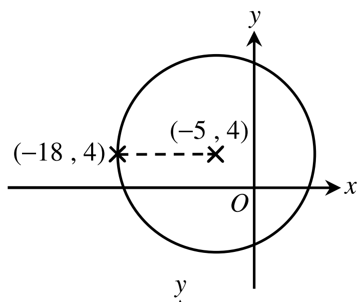
18. The equation $\left(x-\frac{7}{2}\right)^2 + (y+4)^2 = k^2 + 6k + 5$ represents a real circle. Find the range of values of k.

Level 2

19. In the figure, the circle with centre at $(6, 3)$ passes through $(6, -9)$. Find the equation of the circle in the general form.



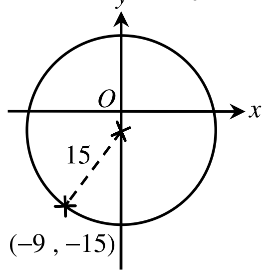
20. In the figure, the circle with centre at $(-5, 4)$ passes through $(-18, 4)$. Find the equation of the circle in the general form.



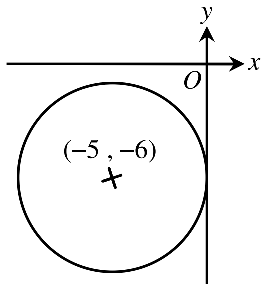
21. In the figure, the radius of the circle is 15 and the centre lies on the y-axis.

(a) Find the coordinates of the centre of the circle.

(b) Find the equation of the circle in the standard form.



✂ **22.** In the figure, the coordinates of the centre of the circle are (−5 , −6). The y-axis is a tangent to the circle.

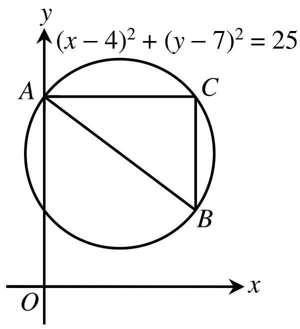


(a) Find the radius of the circle.

(b) Find the equation of the circle in the standard form.

(c) Does $P(-8, -10)$ lie inside, outside or on the circle?

23. In the figure, the equation of the circle is $(x-4)^2+(y-7)^2=25$. The circle passes through A, B and C, where A lies on the y-axis. AB is a diameter of the circle.



(a) Find the coordinates of the centre and the radius of the circle.

(b) Find the coordinates of A and B.

(c) If BC is a vertical line, find the area of $\triangle ABC$.

24. Consider the circle $x^2+y^2-8x+6y-5=0$.

(a) Find the coordinates of the centre and the radius of the circle.

(b) Find the equation of the straight line passing through the centre of the circle and (−3 , 2).

25. Consider the circle $2x^2+2y^2+20x+4y-46=0$.

(a) Find the coordinates of the centre G.

(b) Find the equation of the straight line passing through $B(3, 1)$ and perpendicular to BG.

(c) Find the equation of the straight line passing through $C(-2, 6)$ and parallel to OG, where O is the origin.

26. In each of the following, find the radius of the circle.

(a) $P(6, -8)$ is a point on the circle $x^2+y^2-7x+2ky+6=0$, where k is a constant.

(b) $Q\left(-\frac{1}{2}, -\frac{1}{2}\right)$ is a point on the circle $2x^2+2y^2+kx-10y-5=0$, where k is a constant.

27. Consider the circle $x^2+y^2+12x-30y-28=0$. Determine whether each of the following points lies inside, outside or on the circle.

(a) The origin O **(b)** $A(-14, 0)$ **(c)** $B(2, -2)$

28. It is given that $x^2+y^2-6x+18y-2k+3=0$ is a real circle, where k is a positive constant.

(a) Find the radius of the circle in terms of k.

(b) If the origin lies outside the circle, find the range of values of k.

29. In each of the following, find the range of values of k.

(a) The circle $x^2+y^2+4x-18y+5k=0$ is a real circle.

✂ **(b)** The circle $2x^2+2y^2-6x+3ky+\frac{25}{2}=0$ is an imaginary circle.

✂ **30.** Consider the circle $x^2 + y^2 + kx - 4y - 16 = 0$. If the area of the circle is larger than 120π, find the range of values of k.

31. Consider the circle $x^2 + y^2 + 26x - 18y - 39 = 0$.

(a) Find the coordinates of the centre and the radius of the circle.

(b) If $(-13 , k)$ is a point outside the circle, find the range of values of k in each of the following cases.

(i) $k > 9$ **(ii)** $k < 9$

32. Consider the circle $x^2 + y^2 - 18x - 22y - 23 = 0$.

(a) Find the coordinates of the centre and the radius of the circle.

Explain **(b)** If both $(k , 11)$ and $(9 , k)$ are points inside the circle, how many possible positive integral values of k are there? Explain your answer.

33. Consider the circle $3x^2 + 3y^2 + 15x - 6y - 15 = 0$ and the two points $C(-2 , -2)$ and $D(0 , 3)$.

(a) Determine whether the line segment joining C and D is inside the circle.

(b) Find the equation of the straight line passing through the centre of the circle and perpendicular to CD.

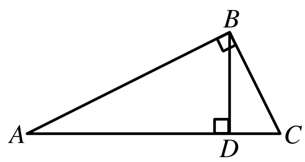
* **34.** Consider the circle C: $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$, where $a > b > 0$.

Explain **(a)** Is C a real circle? Explain your answer.

Explain **(b)** Does $(-2a , 2b)$ lie inside the circle? Explain your answer.

(c) Show that C lies on the left of the y-axis.

* **35.** In the figure, $\triangle ABC$ is a right-angled triangle, where $\angle ABC = 90°$. D is a point on AC such that $BD \perp AC$.



(a) Prove that $\triangle ABD \sim \triangle BCD$.

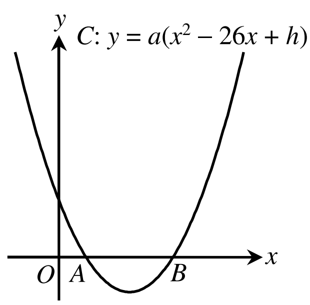
(b) Prove that $BD^2 = AD \times CD$.

(c) A rectangular coordinate system is introduced to the figure so that the coordinates of A and D are $(0 , 0)$ and $(12 , 9)$ respectively, and B lies above the x-axis. It is given that the equation of the circle passing through A, B and D is $2x^2 + 2y^2 - 15x - 30y = 0$.

(i) Find the coordinates of B.

(ii) Using the result of **(b)**, find $AD : CD$.

✂* **36.** In the figure, the curve C: $y = a(x^2 - 26x + h)$ cuts the x-axis at $A(x_1 , 0)$ and $B(x_2 , 0)$, where $x_1 < x_2$, $a > 0$ and $0 < h < 169$.



(a) Find the equation of the perpendicular bisector of AB.

(b) Suppose P is a point on quadrant I such that $AP = BP$. Let C' be the inscribed circle of $\triangle PAB$ with radius k.

(i) Find the coordinates of the in-centre of $\triangle PAB$ in terms of k.

(ii) If $k = 5$ and $AB = 16$, find the equation of C' and $\angle PAB$.

(*Give the answers correct to 3 significant figures if necessary.*)

Answers

Consolidation Exercise 7A

1. **(a)** $x^2 + (y-3)^2 = 4$
   (b) $(x-4)^2 + (y-5)^2 = 36$
   (c) $(x+2)^2 + (y+7)^2 = 25$
   (d) $(x+1)^2 + (y-4)^2 = 5$
   (e) $(x+6)^2 + (y-1)^2 = \frac{1}{16}$
   (f) $(x-5)^2 + (y+2)^2 = \frac{9}{4}$
2. **(a)** $(x+4)^2 + y^2 = 81$
   (b) $(x-6)^2 + (y-3)^2 = 49$
   (c) $(x+3)^2 + (y+2)^2 = 13$
3. **(a)** $x^2 + y^2 = 16$
   (b) $(x-9)^2 + y^2 = 225$
   (c) $x^2 + (y+2)^2 = \frac{49}{4}$
4. **(a)** $(x-8)^2 + (y+4)^2 = 121$
   (b) $(x+6)^2 + (y+3)^2 = 256$
   (c) $(x-5)^2 + (y-7)^2 = 169$
5. **(a)** centre: $(0 , 0)$, radius: 8
   (b) centre: $(6 , 0)$, radius: 2
   (c) centre: $(-1 , 5)$, radius: 3
   (d) centre: $(3 , -2)$, radius: 15
   (e) centre: $(-5 , 0)$, radius: 3
   (f) centre: $(2 , -8)$, radius: $\sqrt{29}$
6. **(a)** $x^2 + y^2 + 6x + 4y - 12 = 0$
   (b) $x^2 + y^2 + 14x - 8y + 49 = 0$
   (c) $x^2 + y^2 - 18x = 0$
   (d) $x^2 + y^2 - 10x + 10y - 10 = 0$
7. **(a)** centre: $(3 , -4)$, radius: 5
   (b) centre: $(2 , 1)$, radius: 2
   (c) centre: $(-6 , -4)$, radius: 9
   (d) centre: $(-5 , 1)$, radius: $\sqrt{21}$
8. **(a)** centre: $(-2 , 3)$, radius: 4
   (b) centre: $(3 , -6)$, radius: 9
   (c) centre: $(2 , 0)$, radius: $\sqrt{11}$
   (d) centre: $(1 , 2)$, radius: $\sqrt{2}$
9. **(a)** a real circle
   (b) a point circle
   (c) an imaginary circle
   (d) a real circle
10. **(a)** centre: $(-1 , -3)$, radius: 4
    (b) yes
11. **(a)** on the circle
    (b) outside the circle
    (c) inside the circle
    (d) outside the circle
12. **(a)** outside the circle
    (b) on the circle
    (c) inside the circle
13. $a < -7$
14. $0 < b < 3$
15. **(a)** centre: $(-3 , 2)$, radius: $\sqrt{17}$
    (b) yes
16. $5, \frac{15}{2}$
17. area: 144π, perimeter: 24π
18. $k < -5$ or $k > -1$
19. $x^2 + y^2 - 12x - 6y - 99 = 0$
20. $x^2 + y^2 + 10x - 8y - 128 = 0$
21. **(a)** $(0 , -3)$
    (b) $x^2 + (y+3)^2 = 225$
22. **(a)** 5
    (b) $(x+5)^2 + (y+6)^2 = 25$
    (c) on the circle
23. **(a)** centre: $(4 , 7)$, radius: 5
    (b) A: $(0 , 10)$, B: $(8 , 4)$
    (c) 24
24. **(a)** centre: $(4 , -3)$, radius: $\sqrt{30}$
    (b) $5x + 7y + 1 = 0$
25. **(a)** $(-5 , -1)$
    (b) $4x + y - 13 = 0$
    (c) $x - 5y + 32 = 0$
26. **(a)** $\sqrt{\frac{89}{4}} \left(\text{or } \frac{\sqrt{89}}{2} \right)$

(b) 3

27. (a) inside the circle

(b) on the circle

(c) outside the circle

28. (a) $\sqrt{2k+87}$

(b) $0 < k < \frac{3}{2}$

29. (a) $k < 17$

(b) $-\frac{8}{3} < k < \frac{8}{3}$

30. $k < -20$ or $k > 20$

31. (a) centre: $(-13 , 9)$, radius: 17

(b) (i) $k > 26$

(ii) $k < -8$

32. (a) centre: $(9 , 11)$, radius: 15

(b) 23

33. (a) yes

(b) $2x + 5y = 0$

34. (a) yes

(b) no

35. (c) (i) $\left(\frac{15}{2}, 15\right)$

(ii) 4 : 1

36. (a) $x = 13$

(b) (i) $(13 , k)$

(ii) C': $x^2 + y^2 - 26x - 10y + 169 = 0$, $\angle PAB = 64.0°$

F5B: Chapter 7B

Date	Task	Progress		
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Book Example 9	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 10	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 11	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 12	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 13	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 14	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Maths Corner Exercise 7B Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	______________ ( )
	Maths Corner Exercise 7B Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	______________ ( )

	Maths Corner Exercise 7B Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: ___________	

5B Lesson Worksheet 7.2

(Refer to Book 5B P.7.19)

Objective: To find the equations of circles from different given conditions.

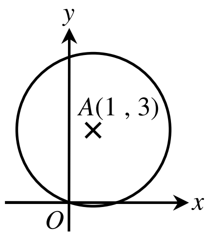
Equations of Circles from Different Given Conditions

(a) If the coordinates of the centre and the radius of a circle are given, we can write down the equation of the circle in the standard form.

(b) In other given conditions, we can find the equation of the circle by setting up simultaneous equations or using the geometric properties of circles.

Instant Example 1

In the figure, the circle with centre $A(1 , 3)$ passes through $(0 , 0)$. Find the equation of the circle in the standard form.



Radius = OA

$= \sqrt{(1-0)^2 + (3-0)^2}$

$= \sqrt{10}$

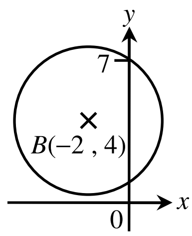
$\therefore$ The equation of the circle is

$(x - 1)^2 + (y - 3)^2 = (\sqrt{10})^2$

<u>$(x - 1)^2 + (y - 3)^2 = 10$</u>

Instant Practice 1

In the figure, find the equation of the circle with centre at B in the standard form.



Radius

= distance between B and (　　,　　)

$= \sqrt{[(\quad) - (\quad)]^2 + [(\quad) - (\quad)]^2}$

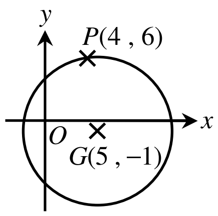
= (　　　)

$\therefore$ The equation of the circle is

$[x - (\quad)]^2 + [y - (\quad)]^2 = (\quad)^2$

____

1.

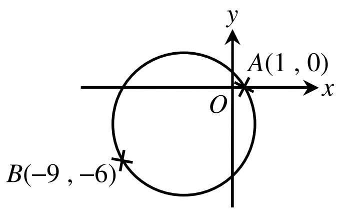


In the figure, the circle with centre at $G(5 , -1)$ passes through $P(4 , 6)$. Find the equation of the circle in the standard form.

Radius = PG

$= \sqrt{(\qquad)^2 + (\qquad)^2}$

2.



In the figure, AB is a diameter of the circle.

(a) Find the coordinates of the centre.

(b) Find the equation of the circle in the standard form.

➥Ex 7B: 1–5

(a) Coordinates of the centre

$= \left(\qquad , \qquad \right)$

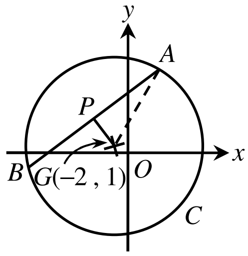
For $P(x_1 , y_1)$ and $Q(x_2 , y_2)$, coordinates of the mid-point of PQ $= \left(\dfrac{x_1 + x_2}{2} , \dfrac{y_1 + y_2}{2} \right)$

(b) Radius of the circle

$= \sqrt{(\qquad)^2 + (\qquad)^2}$

Instant Example 2

In the figure, G is the centre of circle C. $P(-5\ ,\ 5)$ is the mid-point of AB and $AP = PB = 12$. Find the equation of C in the standard form.



$PG = \sqrt{[-2-(-5)]^2+(1-5)^2}$
$= 5$

$\because\ AP = PB$

$\therefore\ PG \perp AB$

$AG^2 = AP^2 + PG^2$

$AG = \sqrt{12^2+5^2}$
$= 13$

Find the radius AG first.

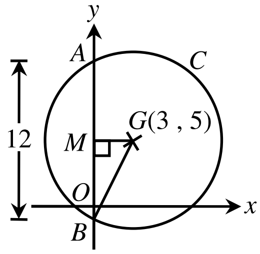
$\therefore$ The equation of C is

$[x-(-2)]^2 + (y-1)^2 = 13^2$

$\underline{(x+2)^2 + (y-1)^2 = 169}$

Instant Practice 2

In the figure, G is the centre of circle C and $MG \perp AB$. Find the equation of C in the standard form.



$\because\ MG \perp (\qquad)$

$\therefore\ MB = \frac{1}{2}(\qquad)$

$= \frac{1}{2} \times (\qquad)$

$= (\qquad)$

$MG = (\qquad)$

$BG^2 = (\qquad)^2 + (\qquad)^2$

$BG = \sqrt{(\qquad)^2+(\qquad)^2} = (\qquad)$

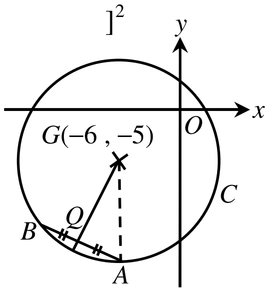
$\therefore$ The equation of C is

$(\qquad)^2 + (\qquad)^2 = (\qquad)^2$

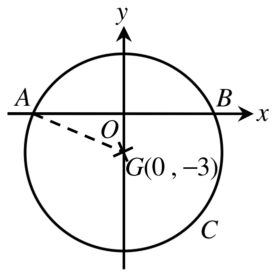
____

3. In the figure, G is the centre of circle C. $Q(-10\ ,\ -13)$ is the mid-point of AB and $AQ = 4$. Find the equation of C in the standard form.

$GQ = \sqrt{[\qquad\qquad]^2+[\qquad\qquad]^2}$
$=$



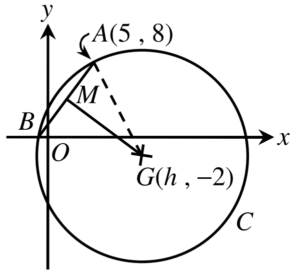
4. In the figure, G is the centre of circle C and $AB = 14$. Find the equation of C in the standard form.



➥Ex 7B: 7, 10, 11

⇪Level Up Question⇪

5. In the figure, G is the centre of circle C. $M(2\ ,\ 4)$ is the mid-point of chord AB. Find the equation of C in the standard form.



7 Equations of Circles

Consolidation Exercise 7B

[*In this exercise, leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*]

Level 1

1. Find the equation of each of the following circles in the standard form.

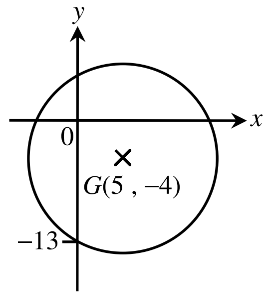
(a) Centre is at (–3 , 6) and diameter is 6.

(b) Centre is at (7 , –6) and diameter is 16.

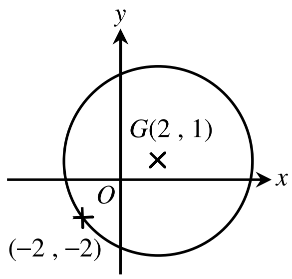
(c) Centre is at (–4 , –5) and diameter is 7.

2. In each of the following figures, *G* is the centre. Find the equation of each circle in the standard form.

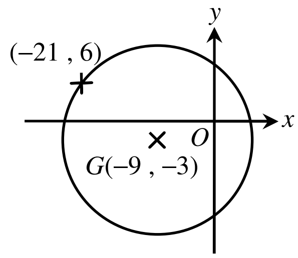
(a)



(b)



(c)



3. Find the equation of each of the following circles in the standard form.

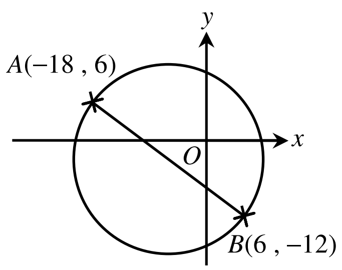
(a) A circle with centre at (1 , 4) passes through (–1 , 8).

(b) A circle with centre at (–5 , –17) passes through (7 , –1).

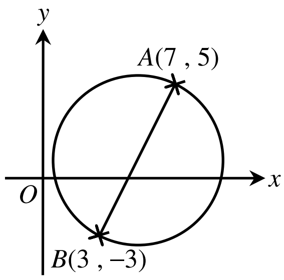
(c) A circle with centre at (–6 , 13) intersects the *y*-axis at (0 , 5).

In each of the following figures, *AB* is a diameter of the circle. Find the equation of each circle in the standard form. **[Nos. 4–5]**

4.



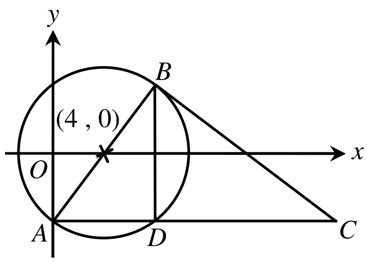
5.



6. In the figure, *AB* is a diameter of the circle and the centre is at (4 , 0). *ADC* is a horizontal line. *BC* = 25 and *CD* = 20.

(a) Find the length of *AB*.

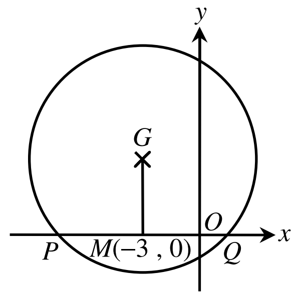
(b) Find the equation of the circle in the standard form.



7. In the figure, G is the centre of the circle. $M(-3, 0)$ is the mid-point of chord PQ. $GM = 4$ and $PQ = 9$.

(a) Find the lengths of PM and PG.

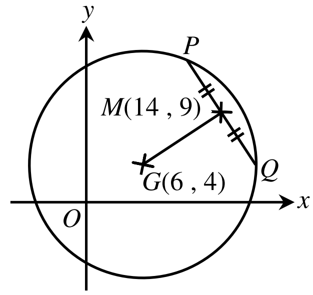
(b) Find the equation of the circle in the standard form.



8. In the figure, $G(6, 4)$ is the centre of the circle. $M(14, 9)$ is the mid-point of chord PQ and $PM = 5$.

(a) Find the lengths of GM and PG.

(b) Find the equation of the circle in the standard form.



9. A circle with centre at $(3, -2)$ passes through $A(-5, 4)$.

(a) Find the equation of the circle in the standard form.

(b) Does $B(1, 7)$ lie inside or outside the circle?

10. $P(-7, 10)$ and $Q(3, -4)$ are the end points of a diameter of a circle.

(a) Find the equation of the circle in the standard form.

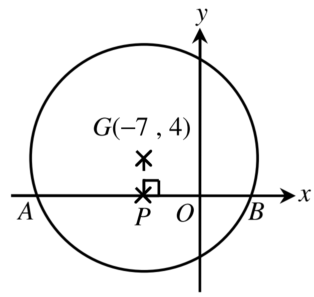
(b) Determine whether each of the following points lies inside, outside or on the circle.

(i) $R(-9, 9)$ **(ii)** $S(5, -2)$

11. In the figure, the centre of the circle is at $G(-7, 4)$. The circle intersects the x-axis at A and B, where $AB = 18$. P is a point on AB such that $GP \perp AB$.

(a) Find the equation of the circle in the standard form.

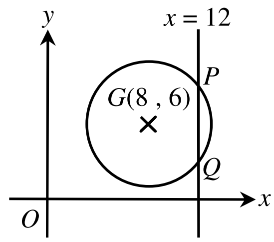
(b) Does $C(-4, 13)$ lie inside the circle?



12. In the figure, $G(8, 6)$ is the centre. The straight line $x = 12$ cuts the circle at P and Q, where $PQ = 6$.

(a) Find the equation of the circle in the standard form.

(b) Is $(5, 10)$ a point on the circle?



✂ **13.** The centre of circle C lies in quadrant IV. C touches the negative y-axis, and touches the positive x-axis at $(13, 0)$.

(a) Write down the coordinates of the centre of C.

(b) Find the equation of the circle in the standard form.

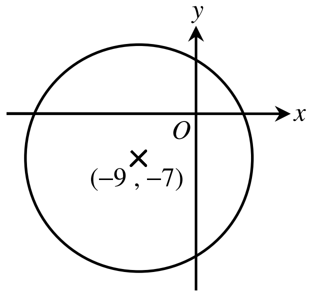
Level 2

14. The centre of a circle is at $(-3 , 4)$ and the diameter is 20.

(a) Find the equation of the circle in the standard form and then convert it into the general form.

(b) If $(k , -2)$ lies on the circle, find all the possible values of k.

15. In the figure, the centre of the circle is at $(-9 , -7)$ and the area is 225π.



(a) Find the radius of the circle.

(b) Write down the equation of the circle in the standard form and then convert it into the general form.

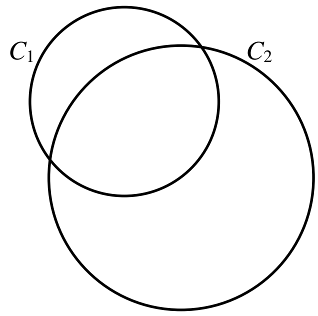
Explain **(c)** If $(-18 , k)$ lies on the circle, where $k > 0$, does $(-3k , 7)$ lie inside the circle? Explain your answer.

16. The centre of circle C_1 is at $(4 , -2)$. The equation of circle C_2 is $4x^2 + 4y^2 - 32x + 16y - 20 = 0$.

Explain **(a)** Are C_1 and C_2 two concentric circles? Explain your answer.

(b) If the radius of C_1 is 4 times that of C_2, find the equation of C_1 in the general form.

17. In the figure, the centre of circle C_1 is at $(-2 , 2)$. The equation of circle C_2 is $x^2 + y^2 - 2x + 4y - 44 = 0$. The radius of C_1 is shorter than that of C_2 by 2 units.

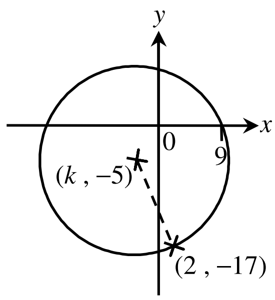


(a) Find the equation of C_1 in the general form.

(b) Determine whether C_1 passes through the centre of C_2.

(c) Does $(3 , 5)$ lie outside both C_1 and C_2?

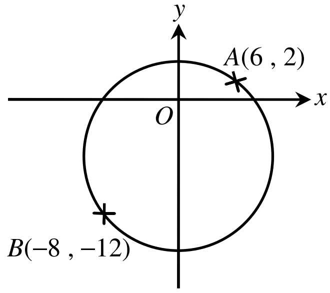
18. In the figure, the circle passes through $(2 , -17)$ and intersects the positive x-axis at $(9 , 0)$. The coordinates of the centre of the circle are $(k , -5)$.



(a) Find the value of k.

(b) Find the equation of the circle in the general form.

19. In the figure, the circle passes through two points $A(6 , 2)$ and $B(-8 , -12)$. Its centre lies on the y-axis.



(a) Find the coordinates of the centre of the circle.

(b) Find the radius of the circle.

Explain **(c)** Is AB a diameter of the circle? Explain your answer.

(d) Find the equation of the circle in the general form.

20. In each of the following, find the equation of the circle passing through A, B and C in the general form.

(a) $A(0 , 0)$, $B(0 , 2)$, $C(8 , 0)$

(b) $A(9 , -1)$, $B(4 , 4)$, $C(4 , -2)$

(c) $A(-3 , -2)$, $B(-4 , -5)$, $C(1 , 0)$

21. The vertices of a triangle are $A(-1 , 11)$, $B(13 , 13)$ and $C(15 , -1)$.

Explain **(a)** Is $\triangle ABC$ a right-angled triangle? Explain your answer.

(b) Find the equation of the circumcircle of $\triangle ABC$ in the general form.

Explain **(c)** If the coordinates of D are $(1 , -3)$, do A, B, C and D lie on the same circle? Explain your answer.

22. $P(-8 , -3)$, $Q(0 , -7)$ and $R(8 , 9)$ are three points in a rectangular coordinate plane.

(a) Find the equation of the circle passing through P, Q and R in the general form.

(b) The coordinates of S are $(k , 3)$. P, Q, R and S are the vertices of a cyclic quadrilateral.

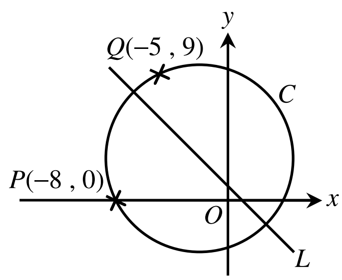
(i) Find the possible values of k.

(ii) For each value of k obtained in **(b)(i)**, find the equation of circle C with PS as diameter in the general form.

23. In the figure, circle C passes through two points $P(-8 , 0)$ and $Q(-5 , 9)$. The centre of C lies on the straight line L: $x + y = 1$.

(a) Find the coordinates of the centre of C.

(b) Find the equation of C in the general form.



24. The slope of a straight line L is -5. L passes through $R(3 , -6)$ and cuts the y-axis at a point Q. A circle C passes through two points $P(-4 , 3)$ and Q. The centre G of the circle C lies on L.

(a) Find the equation of L.

(b) **(i)** Find the coordinates of G.

(ii) Find the equation of C in the general form.

(c) Find the area of the minor sector GPQ in terms of π.

✂ **25.** Two circles C_1 and C_2 touch each other internally at $(0 , 6)$. The equation of C_1 is $x^2 + y^2 - 24x - 12y + 36 = 0$. In each of the following given conditions, find the equation of C_2 in the general form.

(a) C_2 passes through the centre of C_1.

(b) The radius of C_1 is 4 times that of C_2.

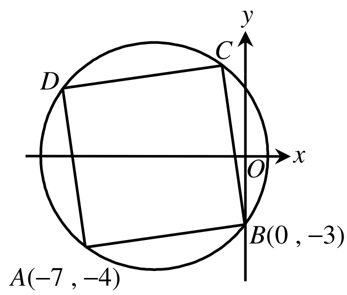
(c) The ratio of the area of C_1 to that of C_2 is $9 : 4$.

26. In the figure, $ABCD$ is a square. The coordinates of A and B are $(-7 , -4)$ and $(0 , -3)$ respectively. The centre of the circumcircle of the square lies on the negative x-axis.

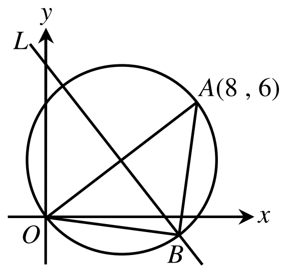
(a) Find the equation of the circumcircle of the square in the general form.

(b) Find the coordinates of C and D.

✂ **(c)** Find the equation of the inscribed circle of the square in the general form.



*27. In the figure, the circle passes through three points $A(8, 6)$, B and O, where $AB = OB$, $AB \perp OB$ and B lies below the x-axis. L is the perpendicular bisector of OA.



(a) Find the equation of the circle in the general form.

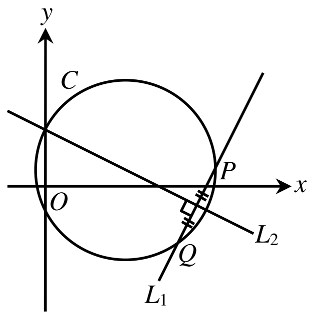
(b) Find the coordinates of B.

✂ (c) L cuts the y-axis at a point C.

(i) Find the coordinates of the orthocentre of $\triangle OAC$.

Explain (ii) Does the in-centre of $\triangle OAC$ lie on L? Explain your answer.

*28. In the figure, the straight line L_1: $2x - y - 34 = 0$ and the circle C intersect at $P(9k, k)$ and $Q(7k, -3k)$.



(a) Find the coordinates of P and Q.

(b) The straight line L_2 is the perpendicular bisector of PQ. The centre G of circle C lies on the left of L_1. The distance between G and the mid-point of PQ is $\sqrt{80}$.

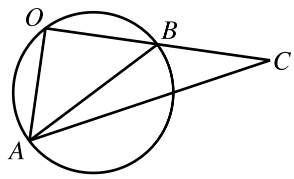
(i) Find the equation of L_2.

(ii) Find the coordinates of G.

(iii) Find the equation of C in the general form.

✂ (c) Another circle C_1 with radius m has the same centre as C, where $0 < m < 10$. Using the section formula, express the coordinates of the point on C_1 which is nearest to Q in terms of m.

*29. In the figure, AB is a diameter of the circle and a median of $\triangle OAC$. It is given that $AC = \sqrt{5}\,OA$.



Explain (a) Is $\triangle OAB$ an isosceles triangle? Explain your answer.

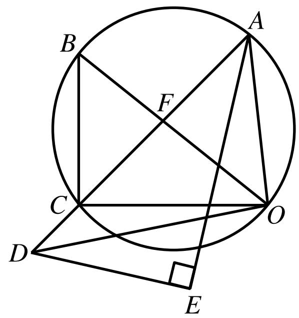
(b) A rectangular coordinate system is introduced to the figure so that the coordinates of O and C are $(0, 0)$ and $(14, -2)$ respectively.

(i) Find the coordinates of A and B.

(ii) Find the equation of the circle.

✂ (iii) Find the coordinates of the circumcentre of $\triangle OAC$.

✂ *30. In the figure, OB is a diameter of the circle. Chord AC cuts OB at F. AC is produced to D such that $\angle ADO = \angle BOC$. E is a point outside the circle such that $AE \perp DE$.



(a) (i) Prove that $\triangle BCO \sim \triangle AOD$.

(ii) Prove that A, D, E and O are concyclic.

(b) A rectangular coordinate system is introduced to the figure so that the coordinates of O, C and D are $(0, 0)$, $(-20, 0)$ and $\left(-\frac{45}{2}, -\frac{5}{2}\right)$ respectively. It is given that $BC = 16$.

(i) Find the coordinates of B.

(ii) Find the equation of the circle $ABCO$.

(iii) Find the equation of the circle $ADEO$.

Answers

Consolidation Exercise 7B

1. **(a)** $(x+3)^2+(y-6)^2=9$

(b) $(x-7)^2+(y+6)^2=64$

(c) $(x+4)^2+(y+5)^2=\frac{49}{4}$

2. **(a)** $(x-5)^2+(y+4)^2=106$

(b) $(x-2)^2+(y-1)^2=25$

(c) $(x+9)^2+(y+3)^2=225$

3. **(a)** $(x-1)^2+(y-4)^2=20$

(b) $(x+5)^2+(y+17)^2=400$

(c) $(x+6)^2+(y-13)^2=100$

4. $(x+6)^2+(y+3)^2=225$

5. $(x-5)^2+(y-1)^2=20$

6. **(a)** 17

(b) $(x-4)^2+y^2=\frac{289}{4}$

7. **(a)** $PM=\frac{9}{2}$, $PG=\sqrt{\frac{145}{4}}\left(\text{or } \frac{\sqrt{145}}{2}\right)$

(b) $(x+3)^2+(y-4)^2=\frac{145}{4}$

8. **(a)** $GM=\sqrt{89}$, $PG=\sqrt{114}$

(b) $(x-6)^2+(y-4)^2=114$

9. **(a)** $(x-3)^2+(y+2)^2=100$

(b) inside the circle

10. **(a)** $(x+2)^2+(y-3)^2=74$

(b) **(i)** outside the circle

(ii) on the circle

11. **(a)** $(x+7)^2+(y-4)^2=97$

(b) yes

12. **(a)** $(x-8)^2+(y-6)^2=25$

(b) yes

13. **(a)** $(13, -13)$

(b) $(x-13)^2+(y+13)^2=169$

14. **(a)** $(x+3)^2+(y-4)^2=100$,
$x^2+y^2+6x-8y-75=0$

(b) $-11, 5$

15. **(a)** 15

(b) $(x+9)^2+(y+7)^2=225$,
$x^2+y^2+18x+14y-95=0$

(c) no

16. **(a)** yes

(b) $x^2+y^2-8x+4y-380=0$

17. **(a)** $x^2+y^2+4x-4y-17=0$

(b) yes

(c) yes

18. **(a)** -3

(b) $x^2+y^2+6x+10y-135=0$

19. **(a)** $(0, -6)$

(b) 10

(c) no

(d) $x^2+y^2+12y-64=0$

20. **(a)** $x^2+y^2-8x-2y=0$

(b) $x^2+y^2-12x-2y+24=0$

(c) $x^2+y^2-2x+10y+1=0$

21. **(a)** yes

(b) $x^2+y^2-14x-10y-26=0$

(c) yes

22. **(a)** $x^2+y^2-6y-91=0$

(b) **(i)** $10, -10$

(ii) when $k=10$: $x^2+y^2-2x-89=0$,
when $k=-10$: $x^2+y^2+18x+71=0$

23. **(a)** $(-2, 3)$

(b) $x^2+y^2+4x-6y-32=0$

24. **(a)** $5x+y-9=0$

(b) **(i)** $(1, 4)$

(ii) $x^2+y^2-2x-8y-9=0$

(c) $\frac{13\pi}{2}$

25. **(a)** $x^2+y^2-12x-12y+36=0$

(b) $x^2+y^2-6x-12y+36=0$

(c) $x^2+y^2-16x-12y+36=0$

26. **(a)** $x^2+y^2+8x-9=0$

(b) C: $(-1, 4)$, D: $(-8, 3)$

(c) $2x^2+2y^2+16x+7=0$

27. **(a)** $x^2+y^2-8x-6y=0$

(b) $(7, -1)$

(c) **(i)** $\left(\frac{7}{4}, 6\right)$

(ii) yes

28. **(a)** P: (18 , 2), Q: (14 , −6)

(b) **(i)** $x + 2y - 12 = 0$

(ii) (8 , 2)

(iii) $x^2 + y^2 - 16x - 4y - 32 = 0$

(c) $\left(\frac{40+3m}{5}, \frac{10-4m}{5}\right)$

29. **(a)** yes

(b) **(i)** A: (−1 , −7), B: (7 , −1)

(ii) $x^2 + y^2 - 6x + 8y = 0$

(iii) $\left(\frac{13}{2}, -\frac{9}{2}\right)$

30. **(b)** **(i)** (−20 , 16)

(ii) $x^2 + y^2 + 20x - 16y = 0$

(iii) $2x^2 + 2y^2 + 49x - 31y = 0$

F5B: Chapter 7C			
Date	Task	Progress	
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)
	Book Example 15	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 16	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 17	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 18	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 19	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 20	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 21	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)

	Maths Corner Exercise 7C Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 7C Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 7C Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: ___________	

5B Lesson Worksheet 7.3A
(Refer to Book 5B P.7.30)

Objective: To understand the possible intersection of a straight line and a circle, and find the number of points of intersection.

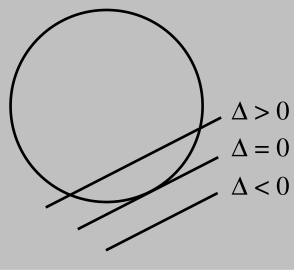
Number of Points of Intersection

Straight line L: $y = mx + c$ .... **(1)**

Circle C: $x^2 + y^2 + Dx + Ey + F = 0$ .... **(2)**

By substituting **(1)** into **(2)**, we can obtain a quadratic equation in x.

Discriminant (Δ) of the quadratic equation	$\Delta > 0$	$\Delta = 0$	$\Delta < 0$
Number of points of intersection	2	1	0



Instant Example 1

Find the number of points of intersection of the straight line L: $2x - y + 3 = 0$ and the circle C: $x^2 + y^2 + 10x + 12 = 0$.

$\begin{cases} 2x - y + 3 = 0 \text{ ....} \textbf{(1)} \\ x^2 + y^2 + 10x + 12 = 0 \text{ ....} \textbf{(2)} \end{cases}$

From **(1)**, $y = 2x + 3$ .... **(3)**

Substitute **(3)** into **(2)**.

$x^2 + (2x + 3)^2 + 10x + 12 = 0$

$x^2 + 4x^2 + 12x + 9 + 10x + 12 = 0$

$5x^2 + 22x + 21 = 0$ .... **(4)**

Discriminant Δ of equation **(4)** $= 22^2 - 4(5)(21)$

$= 64$

> 0

∴ The number of points of intersection is 2.

Instant Practice 1

Find the number of points of intersection of the straight line L: $3x - y - 13 = 0$ and the circle C: $x^2 + y^2 - 2x - 9 = 0$.

$\begin{cases} 3x - y - 13 = 0 \text{ ....} \textbf{(1)} \\ x^2 + y^2 - 2x - 9 = 0 \text{ ....} \textbf{(2)} \end{cases}$

From **(1)**, $y =$ ____________ .... **(3)**

Substitute **(   )** into **(2)**.

$x^2 + ($ $)^2 - 2x - 9 = 0$

____

____

____________________________ .... **(4)**

Discriminant Δ of equation **(4)** = (   )^2 − 4(   )(   )

= (   )

∴ The number of points of intersection is (   ).

Find the number of points of intersection of circle C and straight line L in each of the following. **[Nos. 1–2]**

1. C: $x^2 + y^2 + 14x + 12y + 17 = 0$, L: $y = -4x$

$\begin{cases} y = -4x \text{ ....} \textbf{(1)} \\ x^2 + y^2 + 14x + 12y + 17 = 0 \text{ ....} \textbf{(2)} \end{cases}$

2. C: $x^2 + y^2 + 9y + 3 = 0$, L: $x - 2y + 7 = 0$

➥Ex 7C: 1–4

Make x the subject of the equation of L.

Instant Example 2

If the straight line L: $y = x + 2$ and the circle C: $x^2 + y^2 + 6y + k = 0$ intersect at two points, find the range of values of k.

$\begin{cases} y = x + 2 \text{ ....}\textbf{(1)} \\ x^2 + y^2 + 6y + k = 0 \text{ ....}\textbf{(2)} \end{cases}$

Substitute **(1)** into **(2)**.

$x^2 + (x + 2)^2 + 6(x + 2) + k = 0$

$x^2 + x^2 + 4x + 4 + 6x + 12 + k = 0$

$2x^2 + 10x + 16 + k = 0$ .... **(3)**

∵ L and C intersect at two points.

∴ Discriminant Δ of equation **(3)** > 0

$10^2 - 4(2)(16 + k) > 0$

$100 - 128 - 8k > 0$

$-8k > 28$

$k < -\frac{7}{2}$

Note the change of direction of the inequality sign.

Instant Practice 2

If the straight line L: $y = x - 1$ and the circle C: $x^2 + y^2 - 2x + k = 0$ intersect at two points, find the range of values of k.

$\begin{cases} y = x - 1 \text{ ....}\textbf{(1)} \\ x^2 + y^2 - 2x + k = 0 \text{ ....}\textbf{(2)} \end{cases}$

Substitute **(1)** into **(2)**.

$x^2 + (\qquad)^2 - 2x + k = 0$

____

________________________ .... **(3)**

∵ L and C intersect at two points.

∴ Discriminant Δ of equation **(3)** > 0

$(\quad)^2 - 4(\quad)(\qquad) > 0$

$(\quad) - (\quad) - (\quad)k > 0$

$(\quad)k > (\quad)$

$k < (\quad)$

3. If the straight line L: $y = x + 3$ and the circle C: $x^2 + y^2 + kx - 7 = 0$ intersect at one point, find the values of k.

$\begin{cases} y = x + 3 \text{ ....}\textbf{(1)} \\ x^2 + y^2 + kx - 7 = 0 \text{ ....}\textbf{(2)} \end{cases}$

4. If the number of points of intersection of the straight line L: $y = x - 2$ and the circle C: $x^2 + y^2 - 8y + k = 0$ is at least one, find the range of values of k. ➥Ex 7C: 18, 19

Intersect at:
(a) 2 points (Δ > 0)
(b) 1 point (Δ = 0)
(c) 0 point (Δ < 0)
(d) at least 1 point (Δ ≥ 0)

⇪Level Up Question⇪

5. Explain The circle $x^2 + y^2 + kx + 7y + 4 = 0$ does not intersect the x-axis. Alan claims that the smallest value of k is −4. Do you agree? Explain your answer.

5B Lesson Worksheet 7.3B

(Refer to Book 5B P.7.33)

Objective: To find the coordinates of the points of intersection of a straight line and a circle.

Coordinates of the Points of Intersection

To find the coordinates of the points of intersection, we can solve the simultaneous equations representing the straight line L and the circle C.

Instant Example 1

Find the coordinates of the points of intersection of circle C: $x^2 + y^2 - 4x - 1 = 0$ and straight line L: $y = x - 1$.

$$\begin{cases} x^2 + y^2 - 4x - 1 = 0 \text{ ....(1)} \\ y = x - 1 \text{ ....(2)} \end{cases}$$

Substitute **(2)** into **(1)**.

$$x^2 + (x - 1)^2 - 4x - 1 = 0$$
$$x^2 + x^2 - 2x + 1 - 4x - 1 = 0$$
$$2x^2 - 6x = 0$$
$$x^2 - 3x = 0$$
$$x(x - 3) = 0$$
$$x = 0 \text{ or } 3$$

When $x = 0$, $y = 0 - 1 = -1$.

When $x = 3$, $y = 3 - 1 = 2$.

$\therefore$ The coordinates of the points of intersection are $(0, -1)$ and $(3, 2)$.

Instant Practice 1

Find the coordinates of the points of intersection of circle C: $x^2 + y^2 + 8y + 11 = 0$ and straight line L: $x = 2y + 13$.

$$\begin{cases} x^2 + y^2 + 8y + 11 = 0 \text{ ....(1)} \\ x = 2y + 13 \text{ ....(2)} \end{cases}$$

Substitute **(   )** into **(   )**.

$$(\qquad\qquad)^2 + y^2 + 8y + 11 = 0$$
$$(\quad)y^2 + (\quad)y + (\quad) + y^2 + 8y + 11 = 0$$
$$(\quad)y^2 + (\quad)y + (\quad) = 0$$
$$y^2 + (\quad)y + (\quad) = 0$$
$$[y + (\quad)]^2 = 0$$
$$y = (\quad)(\qquad)$$

When $y = (\quad)$, $x = 2(\quad) + 13 = (\quad)$.

$\therefore$ The coordinates of the point of intersection are $(\quad , \quad)$.

◄ 'L and C have only one point of intersection' means that L is a tangent to C.

Find the coordinates of the points of intersection of circle C and straight line L in each of the following.

[Nos. 1–2]

➥Ex 7C: 5–8

1. C: $x^2 + y^2 - 8 = 0$, L: $y = x - 4$

$$\begin{cases} x^2 + y^2 - 8 = 0 \text{ ....(1)} \\ y = x - 4 \text{ ....(2)} \end{cases}$$

2. C: $x^2 + y^2 - 2y - 24 = 0$, L: $x = 2y - 12$

3. The radius of circle C with centre at $(0\ ,\ 3)$ is $\sqrt{5}$. The equation of the straight line L is $y = x + 2$.

(a) Find the equation of C in the general form.

(b) Find the coordinates of the points of intersection of L and C.

4. Circle C: $x^2 + y^2 + 4x - 6 = 0$ and straight line L: $x + 3y + 2 = 0$ intersect at P and Q. Find the coordinates of the mid-point of PQ.

First find the coordinates of the points of intersection of C and L.

⇪Level Up Question⇪

5. Circle C: $x^2 + y^2 + 10x - 12y + 41 = 0$ and straight line L intersect at two points. The diameter of C with slope -2 lies on L.

(a) Find the equation of L.

(b) Find the coordinates of the two end points of that diameter.

5B Lesson Worksheet 7.3C

(Refer to Book 5B P.7.34)

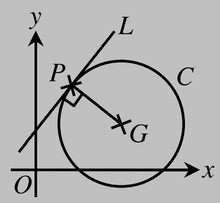
Objective: To find the equations of tangents to a circle.

Equations of Tangents to a Circle

Use the following to find the equation of the tangent L to circle C (with centre at G) at point P.

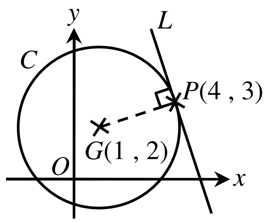
(a) L and the radius PG are perpendicular to each other.

(b) For the quadratic equation in one unknown obtained from the simultaneous equations of L and C, the discriminant $\Delta = 0$.



Instant Example 1

In the figure, G is the centre of circle C. Find the equation of the tangent L to the circle at P.



Slope of $PG = \dfrac{3-2}{4-1}$

$= \dfrac{1}{3}$

Let m be the slope of L.

$\because \quad L \perp PG$ ◄ Tangent ⊥ radius

$\therefore \quad m\left(\dfrac{1}{3}\right) = -1$

$m = -3$

The equation of L is

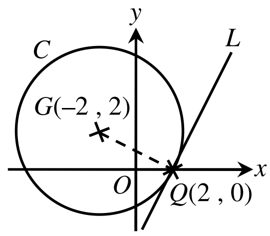
$y - 3 = -3(x - 4)$ ◄ Point-slope form: $y - y_1 = m(x - x_1)$

$y - 3 = -3x + 12$

$\underline{3x + y - 15 = 0}$

Instant Practice 1

In the figure, G is the centre of circle C. Find the equation of the tangent L to the circle at Q.



Slope of $QG = \dfrac{(\quad) - (\quad)}{(\quad) - (\quad)}$

$= (\qquad)$

Let m be the slope of L.

$\because \quad L \perp (\qquad)$

$\therefore \quad m\left(\qquad\right) = -1$

$m = (\quad)$

The equation of L is

$y - (\quad) = (\quad)[x - (\quad)]$

____

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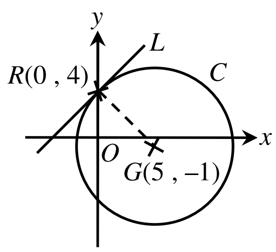
1. In the figure, G is the centre of circle C. Find the equation of the tangent L to the circle at R.

Slope of RG

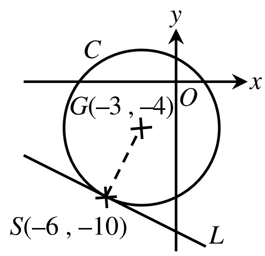
$= \dfrac{(\quad) - (\quad)}{(\quad) - (\quad)}$

$=$



First find the slope of RG. Then find the slope of L.

2. In the figure, G is the centre of circle C. Find the equation of the tangent L to the circle at S.



➥Ex 7C: 10–14

Instant Example 2	***Instant Practice 2***
The straight line $y = -x + c$ touches the circle $x^2 + y^2 - 6x - 23 = 0$. Find the values of c.	The straight line $y = 4x + c$ touches the circle $x^2 + y^2 + 8x - 1 = 0$. Find the values of c.
Substitute $y = -x + c$ into $x^2 + y^2 - 6x - 23 = 0$.	Substitute $y = 4x + c$ into $x^2 + y^2 + 8x - 1 = 0$.
$x^2 + (-x + c)^2 - 6x - 23 = 0$	$x^2 + (\quad)^2 + (\quad)x - (\quad) = 0$
$x^2 + x^2 - 2cx + c^2 - 6x - 23 = 0$	$x^2 + (\quad)x^2 + (\quad)x + (\quad) + (\quad)x - (\quad) = 0$
$2x^2 - (2c + 6)x + c^2 - 23 = 0$	$(\quad)x^2 + (\quad)x + (\quad) - (\quad) = 0$
Since the straight line touches the circle, $\Delta = 0$.	Since the straight line touches the circle, $\Delta = 0$.
$[-(2c + 6)]^2 - 4(2)(c^2 - 23) = 0$	$(\quad)^2 - 4(\quad)(\quad) = 0$
$4c^2 + 24c + 36 - 8c^2 + 184 = 0$	$(\quad) - (\quad) + (\quad) = 0$
$4c^2 - 24c - 220 = 0$	$(\quad)c^2 - (\quad)c - (\quad) = 0$
$c^2 - 6c - 55 = 0$	$c^2 - (\quad)c - (\quad) = 0$
$(c + 5)(c - 11) = 0$	$(\quad)(\quad) = 0$
$c = \underline{\underline{-5}}$ *or* $\underline{\underline{11}}$	$c = \underline{\underline{(\quad)}}$ *or* $\underline{\underline{(\quad)}}$

3. The straight line $y = mx$ touches the circle $x^2 + y^2 - 4y + 2 = 0$. Find the values of m.

Substitute $y = mx$ into $x^2 + y^2 - 4y + 2 = 0$.

$x^2 + (\quad)^2 - 4(\quad) + 2 = 0$

4. The straight line $y = mx - 3$ touches the circle $x^2 + y^2 + 10x = 0$. Find the value of m.

⇪Level Up Question⇪

5. Explain Joan claims that the two straight lines which touch the circle $x^2 + y^2 + 8y + 7 = 0$ with the same y-intercept 1 are perpendicular to each other. Do you agree? Explain your answer.

7 Equations of Circles

✂ Consolidation Exercise 7C

[*When giving answers in this exercise,* **(i)** *express the equations of straight lines in the general form,* **(ii)** *leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*]

Level 1

Without finding the coordinates of the points of intersection, find the number of points of intersection of circle C and straight line L in each of the following. **[Nos. 1–4]**

1. C: $x^2 + y^2 = 24$, L: $x = 5$

2. C: $(x - 2)^2 + (y - 3)^2 = 16$, L: $x + y = 1$

3. C: $x^2 + y^2 + 2x - 4y - 13 = 0$, L: $y = x - 3$

4. C: $x^2 + y^2 + 6x - 9 = 0$, L: $2x + y + 5 = 0$

Find the coordinates of the points of intersection of circle C and straight line L in each of the following. **[Nos. 5–10]**

5. C: $x^2 + y^2 = 8$, L: $y = x$

6. C: $x^2 + y^2 = 34$, L: $y = x - 2$

7. C: $(x - 4)^2 + y^2 = 9$, L: $y = -3$

8. C: $(x - 1)^2 + (y + 2)^2 = 4$, L: $x = 2y$

9. C: $x^2 + y^2 + 4x - 6y + 3 = 0$, L: $y = 3x - 5$

10. C: $x^2 + y^2 + 7y - 9 = 0$, L: $y - 2x + 1 = 0$

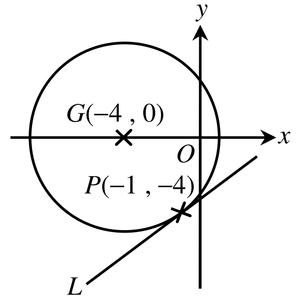
In each of the following, determine whether the straight line L is a tangent to the circle C. **[Nos. 11–12]**

11. C: $x^2 + y^2 = 32$, L: $y = x + 8$

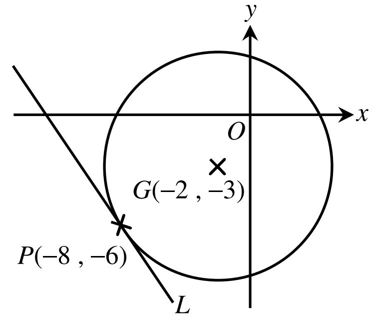
12. C: $x^2 + y^2 + 2x = 0$, L: $x = 3y$

In each of the following, G is the centre and P is a point on the circle. Find the equation of the tangent L to each circle at P. **[Nos. 13–15]**

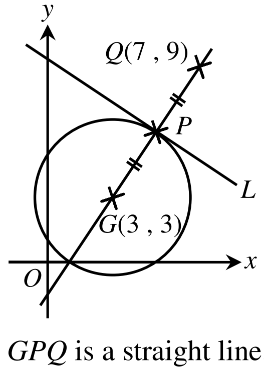
13.



14.



15.



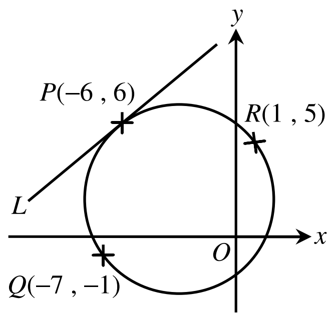
GPQ is a straight line.
P is the mid-point of GQ.

16. The equation of a circle with centre G is $x^2 + y^2 - 8x + 2y + 15 = 0$.

Explain **(a)** Does $A(3 , -2)$ lie on the circle? Explain your answer.

(b) Find the equation of the tangent to the circle at A.

17. In the figure, $Q(-7, -1)$ and $R(1, 5)$ are the end points of a diameter of the circle. The straight line L touches the circle at $P(-6, 6)$. Find the equation of L.



18. The circle C: $(x-7)^2+(y-3)^2=18$ cuts the x-axis at $P(k_1, 0)$ and $Q(k_2, 0)$, where $k_1 < k_2$. Denote the centre of C by G.

(a) Find the coordinates of P and Q.

(b) If the straight lines L_1 and L_2 are the tangents to C at P and Q respectively, find the equations of L_1 and L_2.

19. The straight line L: $y = kx - 3$ touches the circle C: $(x+2)^2 + y^2 = 13$. Find the value of k.

20. The straight line L: $y = 2x + k$ touches the circle C: $x^2 + y^2 - 6x + 4y - 7 = 0$. Find the values of k.

21. The straight line L: $y = x - 7$ is a tangent to the circle C: $x^2 + y^2 + 8x + 6y - k = 0$, where $k > 0$. Find the value of k.

Level 2

22. In each of the following, the straight line L and the circle C do not intersect. Find the range of values of k.

(a) C: $x^2 + y^2 + kx - 4y + 2 = 0$, L: $x + y - 4 = 0$

(b) C: $x^2 + y^2 - 3x + 6y + 10 = 0$, L: $x - 2y + k = 0$

23. In each of the following, the straight line L and the circle C intersect at two points. Find the range of values of k.

(a) C: $(x-3)^2 + y^2 = 8$, L: $x - y + k = 0$

(b) C: $x^2 + y^2 + 6x - ky - 2 = 0$, L: $2x + y - 1 = 0$

24. In each of the following, find the number of points of intersection of the circle $x^2 + y^2 + 4x - 6y + 8 = 0$ and the straight line $2x - y + k = 0$.

(a) $k < 0$

(b) $3 < k < 6$

25. The centre of a circle is at $(-5, 1)$ and the radius is $\sqrt{7}$.

(a) Find the equation of the circle.

(b) Find the number of points of intersection of the straight line $3x + y + 1 = 0$ and the circle.

26. The centre of the circle C is at $(5 , 4)$ and the area of the circle C is 8π. The equation of the straight line L is $x + y - 9 = 0$.

(a) Find the equation of C.

(b) Find the coordinates of the points of intersection of L and C.

Explain **(c)** Does L divide C into two equal parts? Explain your answer.

27. The straight line L: $x - y - k = 0$ touches the circle C: $x^2 + y^2 + 6x + 1 = 0$.

(a) Find the values of k.

(b) Find the two possible equations of L.

28. The straight line L: $x = my + 4$ is a tangent to the circle C: $x^2 + y^2 - 2x + 2my = 0$, where $m > 0$.

(a) Find the value of m.

(b) Find the equation of L.

(c) Find the coordinates of the point of intersection of L and C.

29. The equation of a circle is $x^2 + y^2 + 8x - 6y + 5 = 0$. If the slope of the straight line L that touches the circle is 2, find the two possible equations of L.

30. Circle C passes through three points $(-1 , -1)$, $(11 , 1)$ and $(4 , 6)$.

(a) Find the equation of C.

(b) Two straight lines L_1 and L_2 pass through $A(0 , 7)$. L_1 has a positive slope while L_2 has a negative slope. L_1 and L_2 touch C at P and Q respectively.

(i) Find the equations of L_1 and L_2.

(ii) Find the coordinates of P and Q.

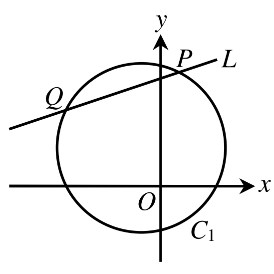
Explain **(iii)** Let R be the centre of C. Is $APRQ$ a square? Explain your answer.

31. Two tangents to circle C: $x^2 + y^2 + 28x - 4y + 100 = 0$ pass through the origin O.

(a) Find the equations of the two tangents.

(b) The two tangents touch C at P and Q respectively, where the x-coordinate of P is less than the x-coordinate of Q. Find the area of $\triangle OPQ$.

32. In the figure, the circle C_1: $4x^2 + 4y^2 + 4x - 8y - 15 = 0$ and the straight line L: $2x - 6y + 17 = 0$ intersect at two points P and Q, where the y-coordinate of P is greater than the y-coordinate of Q.



(a) Find the distance between the centre of C_1 and L.

(b) Find the equation of the circle C_2 with PQ as a diameter.

Explain **(c)** Does the centre of C_1 lie outside C_2? Explain your answer.

33. The coordinates of the centre of the circle C are $(-4 , 3)$. The circumference of C is $8\sqrt{5}\pi$. The y-intercept of the straight line L is k. L is parallel to the straight line $6x = 2y - 1$.

(a) Find the equation of C.

(b) Express the equation of L in terms of k.

(c) L and C intersect at two points A and B.

(i) Express the coordinates of the mid-point of AB in terms of k.

(ii) Hence, if the length of AB is maximum, find the value of k.

34. The equation of the circle C is $x^2 + y^2 - 6x + 10y + 9 = 0$. The equation of the straight line L is $4x - 3y + 23 = 0$. Let P be a point lying on L such that P is nearest to C and R be a point lying on C such that R is nearest to L.

Explain **(a)** Do C and L intersect? Explain your answer.

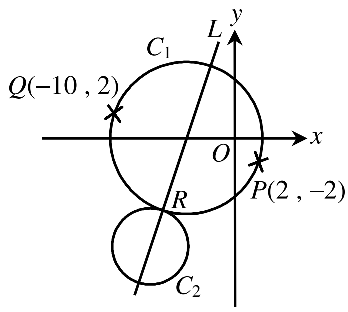
(b) Find the distance between P and R.

(c) Let Q be a point on C that is furthest to R.

(i) Describe the geometric relationship between P, Q and R.

(ii) Find the ratio of the area of $\triangle PQS$ to the area of $\triangle QRS$, where S is any point on L except P.

35. In the figure, $P(2 , -2)$ and $Q(-10 , 2)$ are the end points of a diameter of the circle C_1. The straight line L: $3x - y + p = 0$ passes through the centres of the two circles C_1 and C_2, where C_2 lies in quadrant III. C_1 and C_2 touch each other externally at R. The radius of C_2 is half of that of C_1.



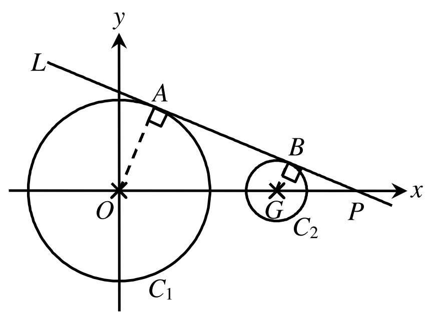
(a) Find the equations of C_1 and C_2.

(b) Find the equation of the common tangent of C_1 and C_2 at R.

(c) **(i)** Show that $D(-4 , -10)$ lies on C_2.

(ii) The tangent to C_2 at D cuts C_1 at two distinct points A and B. Find the coordinates of the mid-point of AB without finding the coordinates of A and B.

36. In the figure, O and G are the centres of circles C_1: $x^2 + y^2 = 225$ and C_2: $(x - 26)^2 + y^2 = 25$ respectively. L is an external common tangent to C_1 and C_2 with points of contact A and B respectively. L cuts the x-axis at P and the slope of L is negative.



(a) By considering similar triangles, find the coordinates of P.

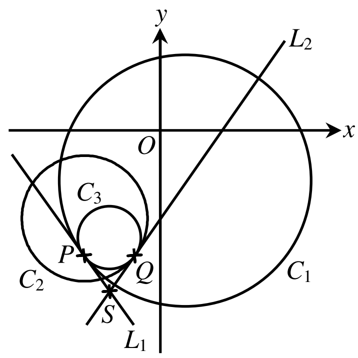
(b) Find the slope of L.

Hence, find the equation of L.

(c) L' is another external common tangent to C_1 and C_2. Find the equation of L'.

Explain **(d)** Do the orthocentres of $\triangle OAP$ and $\triangle GBP$ lie on L? Explain your answer.

37. In the figure, the equations of three circles C_1, C_2 and C_3 are $x^2 + y^2 - 2x + 4y - 20 = 0$, $x^2 + y^2 + 6x + 7y + 15 = 0$ and $2x^2 + 2y^2 + 8x + 17y + 41 = 0$ respectively. C_1 and C_3 touch each other internally at P. C_2 and C_3 touch each other internally at Q. Two straight lines L_1 and L_2 touch C_3 at P and Q respectively. L_1 and L_2 intersect at S.



(a) **(i)** Find the coordinates of the centre and the radius of each circle.

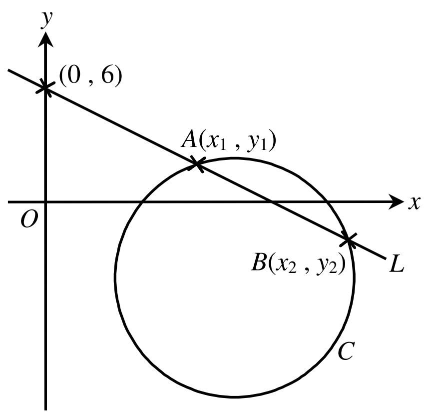
Explain **(ii)** Does the centre of C_2 lie on C_3? Explain your answer.

(b) Suppose the centre of C_3 is at R.

(i) Find the coordinates of P and Q.

(ii) Find the equations of L_1 and L_2.
Hence, find the coordinates of S.

38. In the figure, the circle C: $x^2 + y^2 - 20x + 8y + 76 = 0$ and the straight line L intersect at two points $A(x_1 , y_1)$ and $B(x_2 , y_2)$. L cuts the y-axis at $(0 , 6)$ and its slope is m, where $-1 < m < -\frac{1}{3}$.



(a) Express the equation of L in terms of m.

(b) Show that $(x_1 - x_2)^2 = \dfrac{-80(3m^2 + 10m + 3)}{(1 + m^2)^2}$.

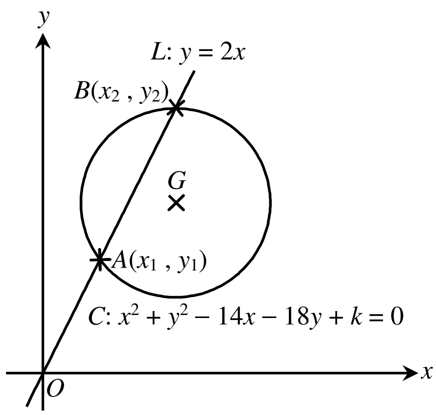
(c) Show that $AB = \sqrt{\dfrac{-80(m + 3)(3m + 1)}{1 + m^2}}$.

(d) Suppose $AB = \sqrt{80}$.

(i) Find the distance between L and the centre of C.

(ii) Find the value of m and the corresponding equation of L.

*** 39.** In the figure, the straight line L: $y = 2x$ passes through the origin and intersects the circle C: $x^2 + y^2 - 14x - 18y + k = 0$ at two points $A(x_1 , y_1)$ and $B(x_2 , y_2)$, where $x_1 < x_2$ and $k < 130$. Let G be the centre of C.



(a) **(i)** Find the coordinates of G.

(ii) Express the radius of C in terms of k.

(b) Show that $x_1 + x_2 = 10$ and $x_1x_2 = \dfrac{k}{5}$.

(c) The length of AB is 4 times the distance between L and G. Let P be a point on L such that P is nearest to G.

(i) Find the value of k.

Explain **(ii)** Does the centroid of $\triangle AGP$ lie on the vertical line which passes through P? Explain your answer.

*__40.__ Consider $P(-5, -7)$ and $Q(5, 13)$. L is the perpendicular bisector of PQ.

(a) Find the equation of L.

(b) Suppose that $G(h, k)$ is a point lying on L. Let C be the circle which is centred at G and passes through P and Q. Prove that the equation of C is $x^2 + y^2 - 2(6 - 2k)x - 2ky + 6k - 134 = 0$.

(c) The coordinates of the point R are $(10, 8)$.

(i) Using the result of **(b)**, find the coordinates of the centre G' of the circle which passes through P, Q and R.

(ii) Find the equation of the tangent to the circle found in **(c)(i)** at R.

Explain **(iii)** Can the radius of C in **(b)** be smaller than the radius of the circle found in **(c)(i)**? Explain your answer.

*__41.__ The equation of circle C with centre G is $x^2 + y^2 - 4x + 6y - 12 = 0$.

(a) Show that $A\left(-\frac{3}{2}, -\frac{7}{2}\right)$ lies inside C and find the equation of the chord with A as the mid-point.

(b) P and Q are the end points of the chord found in **(a)**, where Q lies in quadrant III. Find the coordinates of P and Q.

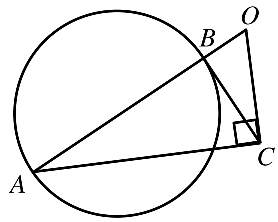
(c) Two straight lines L_1 and L_2 touch C at P and Q found in **(b)** respectively. L_1 and L_2 intersect at R.

(i) Find the coordinates of R. **(ii)** Find the area of $\triangle PQR$.

Explain **(iii)** Is the area of C 8 times that of the inscribed circle of $\triangle PQR$? Explain your answer.

Explain **(iv)** Are the in-centre, the orthocentre, the centroid and the circumcentre of $\triangle PQR$ collinear? Explain your answer.

*__42.__ In the figure, AB is a diameter of the circle and BC is the tangent to the circle at B. AB is produced to O such that $AC \perp CO$.



(a) **(i)** Prove that $\triangle ABC \sim \triangle ACO$.

(ii) Prove that $\triangle ABC \sim \triangle CBO$.

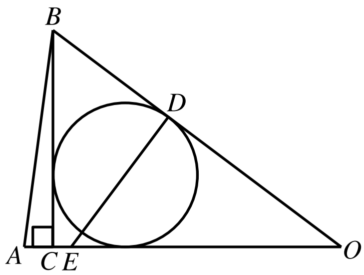
(iii) Prove that $BC = \sqrt{AB \times BO}$.

(b) A rectangular coordinate system is introduced to the figure so that the coordinates of O and A are $(0, 0)$ and $(-15, -10)$ respectively. The equation of BC is $3x + 2y + 13 = 0$.

(i) Find the coordinates of B and C. **(ii)** Find the equation of the circle.

(iii) Find the equation of another tangent to the circle where the tangent passes through C.

*__43.__ The figure shows $\triangle OAB$. C is a point on OA such that $BC \perp OA$. OB touches the inscribed circle of $\triangle OBC$ at D. DE passes through the centre of the inscribed circle of $\triangle OBC$, where E lies on OA.



(a) Prove that $BCED$ is a cyclic quadrilateral.

(b) A rectangular coordinate system is introduced to the figure so that the coordinates of O and C are $(0, 0)$ and $(-20, 0)$ respectively. It is given that $AB = \sqrt{229}$ and $OB = 25$.

(i) Find the coordinates of A and B. **(ii)** Find the equation of the inscribed circle.

(iii) Find the equation of DE.

(iv) Find the equation of the circle passing through B, C, D and E.

Answers

Consolidation Exercise 7C

1. 0

2. 2

3. 1

4. 2

5. (2 , 2), (−2 , −2)

6. (−3 , −5), (5 , 3)

7. (4 , −3)

8. no points of intersection

9. no points of intersection

10. (−3 , −7), (1 , 1)

11. yes

12. no

13. $3x - 4y - 13 = 0$

14. $2x + y + 22 = 0$

15. $2x + 3y - 28 = 0$

16. **(a)** yes

(b) $x + y - 1 = 0$

17. $3x - 4y + 42 = 0$

18. **(a)** $P(4 , 0)$, $Q(10 , 0)$

(b) L_1: $x + y - 4 = 0$, L_2: $x - y - 10 = 0$

19. $\frac{2}{3}$

20. −18, 2

21. 7

22. **(a)** $0 < k < 8$

(b) $k < -10$ or $k > -5$

23. **(a)** $-7 < k < 1$

(b) $k < -6$ or $k > -1$

24. **(a)** 0

(b) 2

25. **(a)** $x^2 + y^2 + 10x - 2y + 19 = 0$

(b) 0

26. **(a)** $x^2 + y^2 - 10x - 8y + 33 = 0$

(b) (3 , 6), (7 , 2)

(c) yes

27. **(a)** −7, 1

(b) $x - y + 7 = 0$, $x - y - 1 = 0$

28. **(a)** 1

(b) $x - y - 4 = 0$

(c) (2 , −2)

29. $2x - y + 1 = 0$, $2x - y + 21 = 0$

30. **(a)** $x^2 + y^2 - 10x - 12 = 0$

(b) **(i)** L_1: $6x - y + 7 = 0$, L_2: $x + 6y - 42 = 0$

(ii) $P(-1 , 1)$, $Q(6 , 6)$

(iii) yes

31. **(a)** $4x + 3y = 0$, $3x - 4y = 0$

(b) 50

32. **(a)** $\sqrt{\frac{5}{2}}\left(\text{or } \frac{\sqrt{10}}{2}\right)$

(b) $4x^2 + 4y^2 + 8x - 20y + 19 = 0$

(c) no

33. **(a)** $x^2 + y^2 + 8x - 6y - 55 = 0$

(b) $3x - y + k = 0$

(c) **(i)** $\left(\frac{5-3k}{10}, \frac{15+k}{10}\right)$

(ii) 15

34. **(a)** no

(b) 5

(c) **(i)** P, Q and R are collinear.

(ii) 3 : 2

35. **(a)** C_1: $x^2 + y^2 + 8x - 24 = 0$,
C_2: $x^2 + y^2 + 14x + 18y + 120 = 0$

(b) $x + 3y + 24 = 0$

(c) **(ii)** (−1 , −1)

36. **(a)** (39 , 0)

(b) slope of L: $-\frac{5}{12}$, L: $5x + 12y - 195 = 0$

(c) $5x - 12y - 195 = 0$

(d) yes

37. (a) (i) centre of C_1: $(1 , -2)$,

radius of C_1: 5,

centre of C_2: $\left(-3 , -\frac{7}{2}\right)$,

radius of C_2: $\frac{5}{2}$,

centre of C_3: $\left(-2 , -\frac{17}{4}\right)$,

radius of C_3: $\frac{5}{4}$

(ii) yes

(b) (i) $P(-3 , -5)$, $Q(-1 , -5)$

(ii) L_1: $4x + 3y + 27 = 0$,

L_2: $4x - 3y - 11 = 0$, $S\left(-2 , -\frac{19}{3}\right)$

38. (a) $mx - y + 6 = 0$

(d) (i) $\sqrt{20}$ (or $2\sqrt{5}$)

(ii) $m = -\frac{1}{2}$, L: $x + 2y - 12 = 0$

39. (a) (i) $(7 , 9)$

(ii) $\sqrt{130 - k}$

(c) (i) 105

(ii) yes

40. (a) $x + 2y - 6 = 0$

(c) (i) $(0 , 3)$

(ii) $2x + y - 28 = 0$

(iii) no

41. (a) $7x + y + 14 = 0$

(b) $P(-2 , 0)$, $Q(-1 , -7)$

(c) (i) $(-5 , -4)$

(ii) $\frac{25}{2}$

(iii) no

(iv) yes

42. (b) (i) $B(-3 , -2)$, $C(1 , -8)$

(ii) $x^2 + y^2 + 18x + 12y + 65 = 0$

(iii) $2x - 3y - 26 = 0$

43. (b) (i) $A(-22 , 0)$, $B(-20 , 15)$

(ii) $x^2 + y^2 + 30x - 10y + 225 = 0$

(iii) $4x - 3y + 75 = 0$

(iv) $4x^2 + 4y^2 + 155x - 60y + 1\ 500 = 0$

F5B: Chapter 8A				
Date	Task	Progress		
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Book Example 1	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 2	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 3	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 4	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 5	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Maths Corner Exercise 8A Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	__________ ( )
	Maths Corner Exercise 8A Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	__________ ( )
	Maths Corner Exercise 8A Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	__________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: ________	

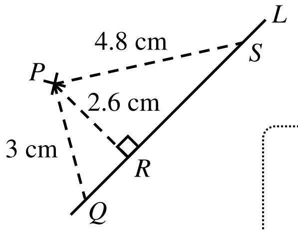
5B Lesson Worksheet 8.0

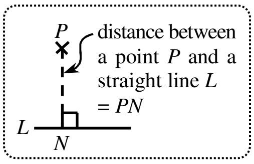
(Refer to Book 5B P.8.3)

Objective: To review the distance between a point and a line, and the distance between two parallel lines.

Distance between a Point and a Line

1. In the figure, find the distance between point P and the straight line L.



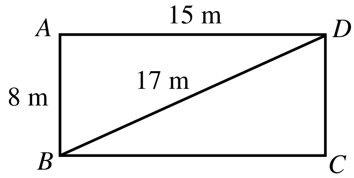


Distance between P and L

= (   )

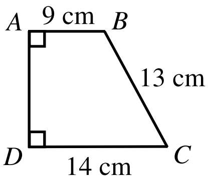
= ________

2. In the figure, $ABCD$ is a rectangle. Find the distance between point D and BC.



➥Review Ex: 1

3. In the figure, $AB \perp AD$ and $AD \perp DC$. Find the distance between B and DC.



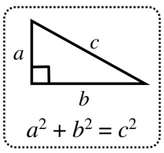
Let E be a point on (   ) such that

(   ) ⊥ (   ).

EC = (   ) − (   )

= [(   ) − (   )] cm

=



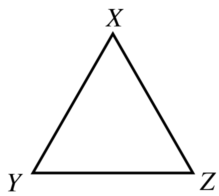
In △ BEC,

$(\quad)^2 + (\quad)^2 = (\quad)^2$

$(\quad) = \sqrt{(\quad)^2 - (\quad)^2}$ cm

=

4. In the figure, △ XYZ is an equilateral triangle of side $2\sqrt{3}$ cm. Find the distance between X and YZ.



Let (   ) be a point on (   ) such that

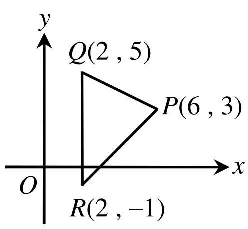
(   ) ⊥ (   ).

$(\quad) = \frac{1}{2}(\quad) = \frac{1}{2} \times (\quad)$ cm

5. In the figure, PQR is a triangle.

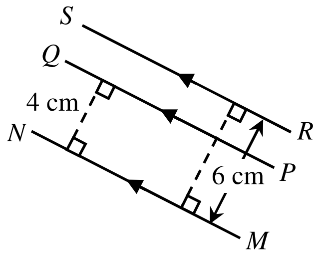
(a) Find the height of △ PQR with QR as the base.

(b) Find the area of △ PQR.



Distance between Two Parallel Lines

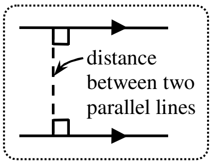
6. In the figure, MN // PQ // RS. Find the distance between PQ and RS.



Distance between PQ and RS

= [(   ) − (   )] cm

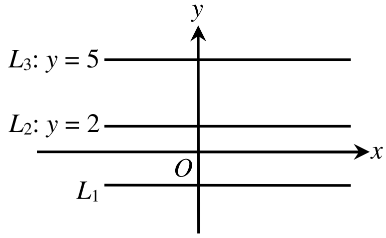
= ________



7. In the figure, straight lines L_1 and L_3: $y = 5$ are equidistant from the straight line L_2: $y = 2$.

(a) Find the distance between L_2 and L_3.

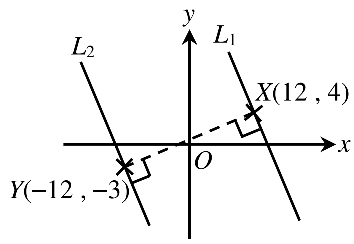
(b) Find the equation of L_1.



➥Review Ex: 2

The equation of a horizontal line passing through (h, k) is $y = k$.

8. In the figure, $X(12, 4)$ and $Y(-12, -3)$ are points on straight lines L_1 and L_2 respectively. If $XY \perp L_1$ and $XY \perp L_2$, find the distance between L_1 and L_2.



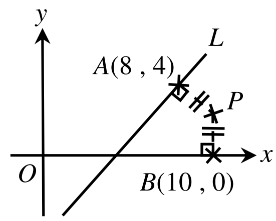
Distance between two points (x_1, y_1) and (x_2, y_2)
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇪Level Up Question⇪

9. In the figure, $A(8, 4)$ and $B(10, 0)$ are points on straight line L and the x-axis respectively. Straight line L_1 passes through $P(a, b)$, where L // L_1, $PA = PB$, $PA \perp L$ and $PB \perp$ the x-axis.

(a) Find the coordinates of P.

(b) Find the distance between L and L_1.



5B Lesson Worksheet 8.2

(Refer to Book 5B P.8.5)

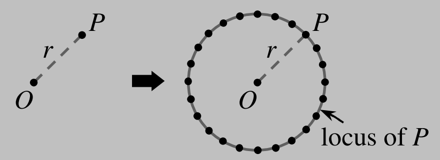
Objective: To describe and sketch the locus of points satisfying given conditions.

Relationship between a Moving Point and Fixed Point(s)

(a) A moving point and a fixed point

Condition: A moving point P maintains a fixed distance of r from a fixed point O.

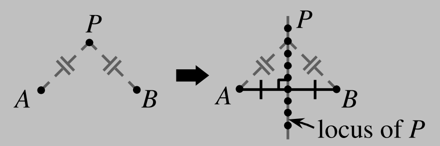
Locus: A circle with centre O and radius r.



(b) A moving point and two fixed points

Condition: A moving point P maintains an equal distance from two fixed points A and B, i.e. $PA = PB$.

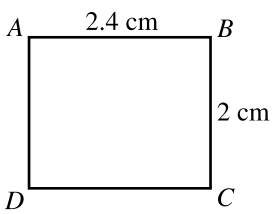
Locus: The perpendicular bisector of the line segment AB.



1. The figure shows a rectangle $ABCD$. A moving point P maintains a fixed distance of 1 cm from D.

(a) Sketch the locus of P.

Step 1: Use a ruler to draw several points which are 1 cm from D.
Step 2: Use a curve to join all the points drawn.



(b) Describe the locus of P.

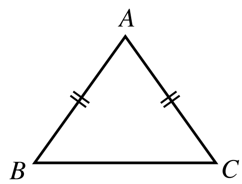
The locus is a (   ) with centre (   ) and radius (   ) cm.

2. In the figure, $AB = AC$. A moving point P maintains an equal distance from B and C, i.e. $PB = PC$.

➥ Ex 8A: 1, 2

(a) Sketch the locus of P.

Step 1: Use a pair of compasses to draw several points such that $PB = PC$.
Step 2: Use a ruler to join all the points drawn.



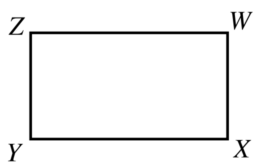
(b) Describe the locus of P.

The locus is the (   ) of the (   ).

Consider rectangle $WXYZ$ as shown in the figure. **[Nos. 3–4]**

3. A moving point P maintains an equal distance from Z and Y, i.e. $PZ = PY$.

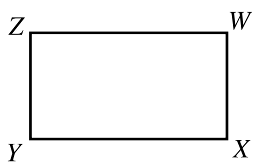
(a) Sketch the locus of P.



(b) Describe the locus of P.

4. A moving point P maintains a fixed distance of 1.2 cm from X.

(a) Sketch the locus of P.



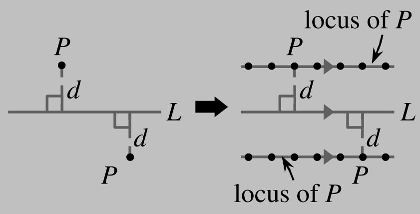
(b) Describe the locus of P.

Relationship between a Moving Point and a Fixed Line or a Fixed Line Segment

(c) A moving point and a line

Condition: A moving point P maintains a fixed distance of d from a fixed line L.

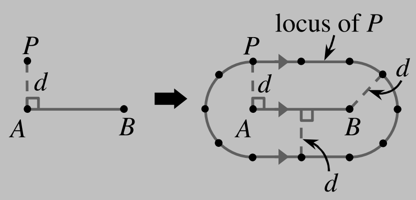
Locus: A pair of parallel lines which are parallel to L, one on either side of L and each at a distance of d from L.



(d) A moving point and a line segment

Condition: A moving point P maintains a fixed distance of d from a fixed line segment AB.

Locus: A closed figure formed by two line segments, which are parallel to AB with the same length as AB and each at a distance of d from AB, and two semi-circles of radii d and with centres A and B respectively.



5. In the figure, a moving point P maintains a fixed distance of 1 cm from the fixed line L.

(a) Sketch the locus of P.

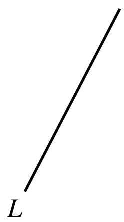
Step 1: On one side of L, mark several points 1 cm from L.
Step 2: On the other side of L, mark several points 1 cm from L.
Step 3: Use a ruler to join the points drawn on each side.

(b) Describe the locus of P.

The locus is a pair of (   ) which are (   ), one on either side of (   ) and each at a distance of (   ) cm from (   ).

6. In the figure, a moving point Q maintains a fixed distance of 1.5 cm from the fixed line L.

(a) Sketch the locus of Q. ➥Ex 8A: 3



(b) Describe the locus of Q.

7. In the figure, a moving point P maintains a fixed distance of 1 cm from the line segment AB. ➥Ex 8A: 6

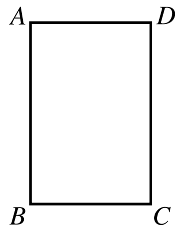
(a) Sketch the locus of P.

(b) Describe the locus of P.

The locus is a (   ) figure formed by (   ) line segments, which are (   ) to AB with the same length as (   ) and each at a distance of (   ) from AB, and two (   ) of radii (   ) and with centres (   ) and (   ) respectively.

Step 1: On the left and right of the line segment AB, draw several points such that the distance of each point from AB is 1 cm.
Step 2: For the region above A, draw several points which are 1 cm from A.
Step 3: For the region below B, draw several points which are 1 cm from B.
Step 4: Join all the points drawn.

8. In the figure, *ABCD* is a rectangle, where $AB = 2.4$ cm and $BC = 1.6$ cm. A moving point *Q* lies outside the rectangle and it maintains a fixed distance of 1.2 cm from the line segment *AD*.



(a) Sketch the locus of *P*.

(b) Describe the locus of *P*.

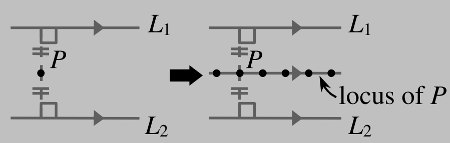
The locus consists of (   ) line segment and (   ) semi-circles outside the rectangle. The line segment has the same length as (   ), parallel to (   ) and at a distance of (   ) from *AD*. The (   ) semi-circles are of radii (   ) and with centres (   ) and (   ) respectively.

Relationship between a Moving Point and Two Fixed Lines

(e) A moving point and two parallel lines

Condition: A moving point *P* maintains an equal distance from two fixed parallel lines L_1 and L_2.

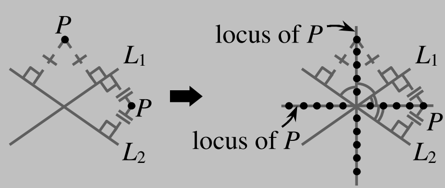
Locus: A straight line parallel to L_1 and L_2, and equidistant from L_1 and L_2.



(f) A moving point and two intersecting lines

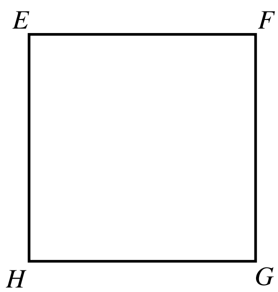
Condition: A moving point *P* maintains an equal distance from two fixed intersecting lines L_1 and L_2.

Locus: A pair of straight lines passing through the point of intersection of L_1 and L_2. They are the two angle bisectors of the angles formed between L_1 and L_2.



9. The figure shows a square *EFGH*. A moving point *P* lies inside the square and it is equidistant from the line segments *EF* and *HG*.

(a) Sketch the locus of *P*.

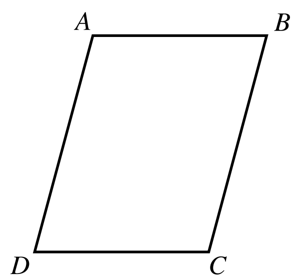


(b) Describe the locus of *P*.

The locus is the (   ) joining the (   ) of (   ) and (   ).

10. The figure shows a parallelogram *ABCD*. A moving point *Q* lies inside the parallelogram and it is equidistant from the line segments *AD* and *BC*.

(a) Sketch the locus of *Q*. ➥Ex 8A: 5

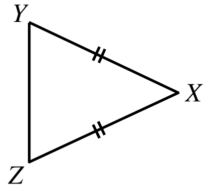


(b) Describe the locus of *Q*.

11. In the figure, $XY = XZ$. A point P moves inside $\triangle XYZ$ and it maintains an equal distance from XY and XZ.

(a) Sketch the locus of P.

Step 1: Use a pair of compasses to draw several points which are equidistant from XY and XZ.
Step 2: Use a ruler to join all the points drawn.

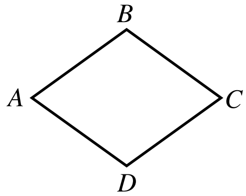


(b) Describe the locus of P.

The locus is the ( ) of ∠( ) inside △( ).

12. The figure shows a rhombus $ABCD$. A point P moves inside the rhombus and it maintains an equal distance from AD and CD. ➥Ex 8A: 4

(a) Sketch the locus of P.



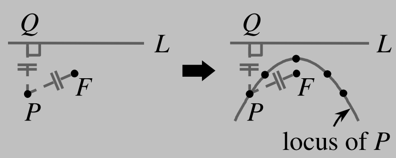
(b) Describe the locus of P.

Relationship between a Moving Point and a Fixed Point and a Fixed Line

(g) A moving point and a fixed point and a fixed line

Condition: A moving point P maintains an equal distance from a fixed point F and a fixed line L, i.e. $PF = PQ$.

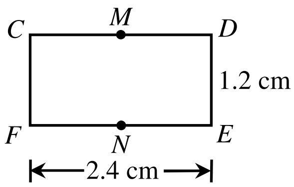
Locus: A parabola.



13. In the figure, $CDEF$ is a rectangle. M and N are the mid-points of CD and FE respectively. A point P moves inside the rectangle and it is equidistant from M and the line segment FE.

(a) Sketch the locus of P.

Step 1: Mark the mid-point of MN. (Note that $CM = CF = DM = DE$.)
Step 2: Draw a parabola from C to D passing through the point obtained in **Step 1**.



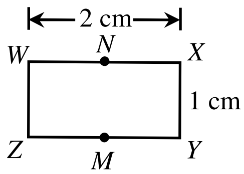
(b) Describe the locus of P.

The locus is a ( ), which lies inside ( ). The ( ) opens ( ) with ( ) and ( ) as the end points.

14. In the figure, $WXYZ$ is a rectangle. M and N are the mid-points of ZY and WX respectively. A point P moves inside the rectangle and it is equidistant from M and the line segment WX. ➥Ex 8A: 8

(a) Sketch the locus of P.

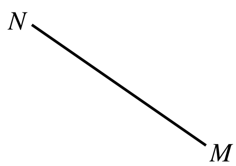
Step 1: Mark the mid-point of MN. (Note that $ZM = ZW = YM = YX$.)
Step 2: Draw a parabola from Z to Y passing through the point obtained in **Step 1**.



(b) Describe the locus of P.

⇪Level Up Question⇪

15. The figure shows a line segment MN of length 2.8 cm. A point P moves such that the area of $\triangle PMN = 1.4\text{ cm}^2$. Sketch and describe the locus of P.

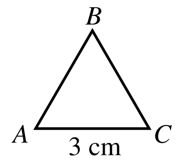


8 Locus

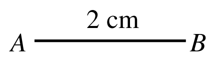
Consolidation Exercise 8A

Level 1

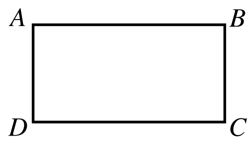
1. The figure shows an equilateral triangle ABC of side 3 cm. A moving point P maintains a fixed distance of 3 cm from C. Sketch and describe the locus of P.



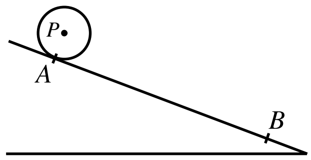
2. In the figure, AB is a line segment of length 2 cm. A moving point P maintains a fixed distance from A and that fixed distance is equal to half of the length of AB. Sketch and describe the locus of P.



3. In the figure, $ABCD$ is a rectangle. A moving point P maintains an equal distance from A and D. Sketch and describe the locus of P.



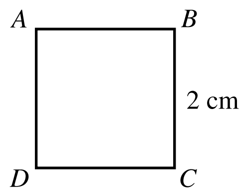
4. In the figure, A and B are points on an inclined plane such that $AB = 10$ cm. A ball with centre P and radius 1 cm rolls down from A to B along the plane. Sketch and describe the locus of P.



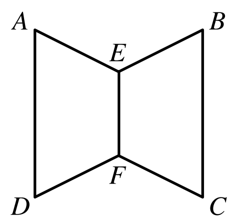
5. In the figure, $ABCD$ is a square of side 2 cm. A point P moves inside the square $ABCD$, and it maintains an equal distance from B and D.

(a) Sketch and describe the locus of P.

(b) Describe the geometric relationship between the locus of P and BD.



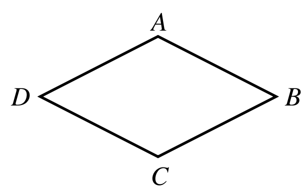
6. The figure on the right is formed by two identical trapeziums $ADFE$ and $BCFE$, where AD // EF // BC. A point P moves inside $AEBCFD$, and it maintains an equal distance from AD and BC. Sketch and describe the locus of P.



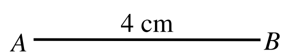
7. In the figure, $ABCD$ is a rhombus. P is a point inside the rhombus. When P moves, it maintains an equal distance from AB and BC.

(a) Sketch and describe the locus of P.

(b) Describe the geometric relationship between the locus of P and $\angle ABC$.



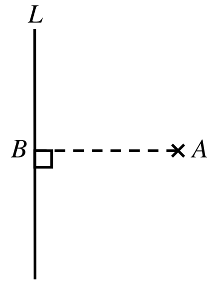
8. In the figure, the length of a line segment AB is 4 cm. A moving point P maintains a fixed distance of 2 cm from the line segment AB. Sketch and describe the locus of P.



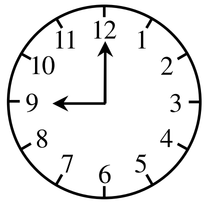
9. In the figure, L is a vertical line and B is a fixed point on L. A is a fixed point on the right of L and $AB \perp L$. A moving point P maintains an equal distance from A and the line L.

(a) Sketch and describe the locus of P.

Explain **(b)** Does the perpendicular bisector of AB intersect the locus of P? Explain your answer.



10. The figure shows a clock. The length of the hour-hand is 15 cm. Let P be the tip of the hour-hand. If it is 9 o'clock now, sketch and describe the locus of P during the next 3 hours.

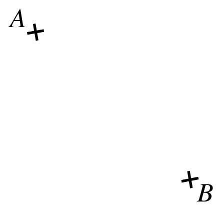


Level 2

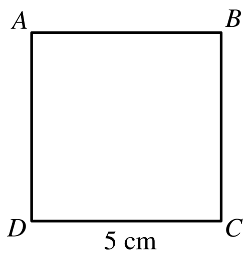
11. In the figure, A and B are two fixed points on a plane.

(a) A moving point P maintains a fixed distance from the mid-point of AB and that fixed distance is equal to half of the length of AB.

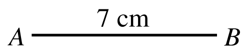
(b) A point Q moves such that $AQ^2 + QB^2 = AB^2$. Sketch and describe the locus of Q.



12. The figure shows a square $ABCD$ of side 5 cm. A point P lies outside the square $ABCD$. The point P moves such that it maintains a fixed distance of 2 cm from the line segment AB. Sketch and describe the locus of P.



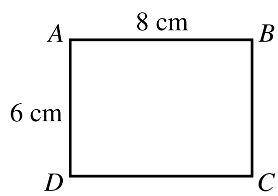
13. In the figure, AB is a line segment of length 7 cm. A point P moves such that the area of $\triangle PAB$ is 14 cm^2. Sketch and describe the locus of P.



14. In the figure, $ABCD$ is a rectangle. AB = 8 cm and AD = 6 cm. A point P moves inside the rectangle such that the area of $\triangle PAD$ is 9 cm^2.

(a) Sketch and describe the locus of P.

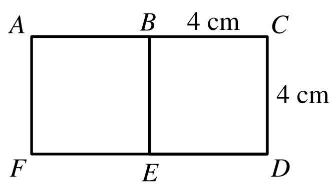
(b) Find the area of $\triangle PBC$.



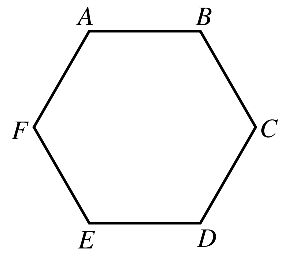
15. In the figure, $ABEF$ and $BCDE$ are two squares of sides 4 cm. A point P moves inside the rectangle $ACDF$, and it is equidistant from E and AC.

(a) Sketch and describe the locus of P.

Explain **(b)** A point Q moves inside the rectangle $ACDF$, and it is equidistant from B and FD. How many points of intersection are there between the locus of P and the locus of Q? Explain your answer.



16. The figure shows a regular hexagon *ABCDEF*.

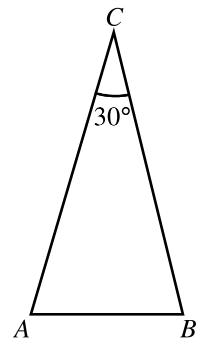


(a) A point *P* moves inside *ABCDEF*, and it maintains an equal distance from *AB* and *BC*. Sketch and describe the locus of *P*.

Explain **(b)** A point *Q* moves inside *ABCDEF*, and it maintains an equal distance from *D* and *F*. John claims that the locus obtained in **(a)** and the locus of *Q* are the same. Do you agree? Explain your answer.

(c) A moving point *R* maintains an equal distance from *D* and *E*. It is given that the locus of *R* and the locus obtained in **(a)** intersect at a point *S*. Describe the geometric relationship between *S* and *ABCDEF*.

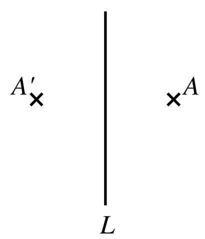
17. In the figure, *AB* is a fixed line segment and $\angle ACB = 30°$. *P* is a point above *AB*. The point *P* moves such that $\angle APB = 30°$.



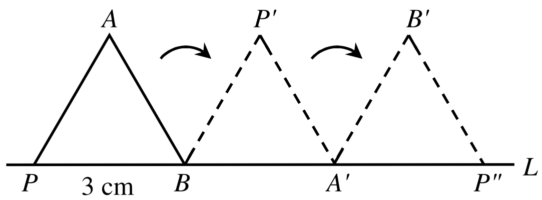
(a) Sketch and describe the locus of *P*.

(b) If *AB* = 4 cm, find the area enclosed by the locus of *P* and the line segment *AB* in terms of π.
(*Leave the radical sign '$\sqrt{\ }$' in the answer.*)

***18.** In the figure, *A'* is the image of *A* when reflected in a straight line *L*. When a point *P* moves, no point on *L* is equidistant from *P* and *A*. Sketch and describe the locus of *P*.



***19.**



In the figure, △*PAB* is an equilateral triangle of side 3 cm. *PB* lies on a straight line *L*. △*PAB* undergoes the following two rotations along *L*, where *A'* and *P''* lie on *L*.

I. △*PAB* rotates clockwise about *B* to become △*BP'A'* first.

II. Then △*BP'A'* rotates clockwise about *A'* to become △*A'B'P''*.

(a) Sketch and describe the locus of *P* during the two rotations.

(b) Find the total distance travelled by *P* during the two rotations in terms of π.

Answers

Consolidation Exercise 8A

1. a circle with centre C and radius 3 cm

2. a circle with centre A and radius 1 cm

3. the perpendicular bisector of the line segment AD

4. a line segment of 10 cm, parallel to AB and at a distance of 1 cm from AB

5. **(a)** line segment AC

(b) The locus of P is perpendicular to BD and bisects BD.

6. line segment EF

7. **(a)** line segment BD

(b) The locus of P is the angle bisector of $\angle ABC$.

8. A closed figure formed by two line segments and two semi-circles. The two line segments are 4 cm long, parallel to AB and at a distance of 2 cm from AB. The two semi-circles are of radii 2 cm and with centres A and B respectively.

9. **(a)** a parabola opening to the right with the mid-point of AB as the vertex

(b) yes

10. $\frac{1}{4}$ of a circle with radius 15 cm

11. **(a)** a circle with AB as a diameter

(b) a circle with AB as a diameter

12. The locus consists of a line segment and two semi-circles outside the square. The line segment is 5 cm long, parallel to AB and at a distance of 2 cm from AB. The two semi-circles are of radii 2 cm and with centres A and B respectively.

13. two straight lines parallel to AB and at a distance of 4 cm from AB

14. **(a)** a line segment of length 6 cm inside $ABCD$, parallel to AD and at a distance of 3 cm from AD

(b) 15 cm^2

15. **(a)** a parabola which lies inside $ACDF$ and opens downward with D and F as the end points

(b) 1

16. **(a)** line segment BE

(b) yes

(c) S is the circumcentre of the hexagon $ABCDEF$.

17. **(a)** The locus of P is an arc passing through A, B and C with the angle at the centre subtended by $\overset{\frown}{ACB}$ equal to 300°, excluding the points A and B.

(b) $\left(\frac{40\pi}{3} + 4\sqrt{3}\right)\text{cm}^2$

18. a straight line passing through A and A', excluding points A and A'

19. **(a)** The locus of P consists of two arcs $\overset{\frown}{PAP'}$ and $\overset{\frown}{P'B'P''}$. The arc $\overset{\frown}{PAP'}$ is of radius 3 cm and with centre B, where $\angle PBP' = 120°$. The arc $\overset{\frown}{P'B'P''}$ is of radius 3 cm and with centre A', where $\angle P'A'P'' = 120°$.

(b) 4π cm

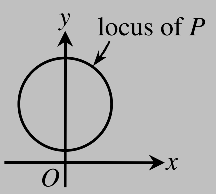
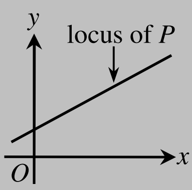
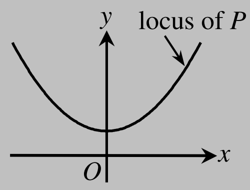
F5B: Chapter 8B				
Date	Task	Progress		
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Book Example 6	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 7	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 8	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 9	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 10	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Maths Corner Exercise 8B Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	________ ( )
	Maths Corner Exercise 8B Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	________ ( )
	Maths Corner Exercise 8B Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: ________	

5B Lesson Worksheet 8.3

(Refer to Book 5B P.8.17)

Objective: To describe the locus of points with algebraic equations, including equations of straight lines, circles and parabolas.

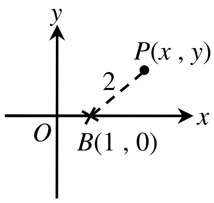
Describing Locus with an Algebraic Equation

The locus is a circle.	*The locus is a straight line.*	*The locus is a parabola.*
$(x-h)^2+(y-k)^2=r^2$	$ax+by+c=0$	$y=ax^2+bx+c$
		

Distance between two points (x_1, y_1) and (x_2, y_2) = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Instant Example 1

In the figure, a moving point $P(x, y)$ maintains a fixed distance of 2 from $B(1, 0)$. Find the equation of the locus of P.



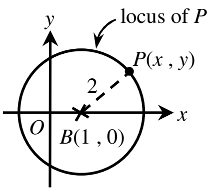
$\because$ $PB = 2$ ◄ The condition for P

$\therefore$ $\sqrt{(x-1)^2+(y-0)^2} = 2$

$(x-1)^2+y^2=4$ ◄ Square both sides.

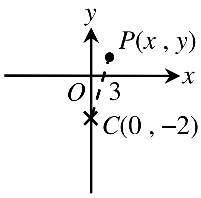
$\therefore$ The equation of the locus of P is $(x-1)^2+y^2=4$.

The locus of P is a circle with centre B and radius 2.



Instant Practice 1

In the figure, a moving point $P(x, y)$ maintains a fixed distance of 3 from $C(0, -2)$. Find the equation of the locus of P.



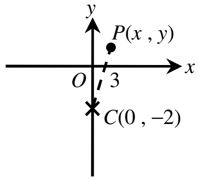
$\because$ $PC = (\quad)$

$\therefore$ $\sqrt{[x-(\quad)]^2+[y-(\quad)]^2} = (\quad)$

$x^2+[y+(\quad)]^2=(\quad)$

$\therefore$ The equation of the locus of P is ____________________.

The locus of P is ________________
________________.



In each of the following, Q is a point in a rectangular coordinate plane. A moving point $P(x, y)$ maintains a fixed distance of d from Q. Find the equation of the locus of P. **[Nos. 1–2]**

1. $Q(4, -1)$ and $d = 5$

$\because$ $PQ = (\quad)$

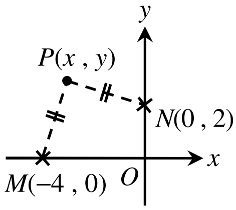
$\therefore$ $\sqrt{[x-(\quad)]^2+[y-(\quad)]^2} = (\quad)$

2. $Q(-8, -3)$ and $d = 6$

➥Ex 8B: 1–4

Instant Example 2

In the figure, $M(-4 , 0)$ and $N(0 , 2)$ are two points in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from M and N, i.e. $PM = PN$. Find the equation of the locus of P.



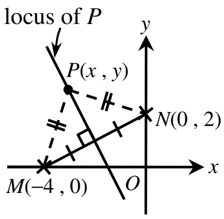
$\because \quad PM = PN$

$\therefore \quad \sqrt{[x-(-4)]^2+(y-0)^2} = \sqrt{(x-0)^2+(y-2)^2}$

$$(x + 4)^2 + y^2 = x^2 + (y - 2)^2$$
$$x^2 + 8x + 16 + y^2 = x^2 + y^2 - 4y + 4$$
$$8x + 4y + 12 = 0$$
$$2x + y + 3 = 0$$

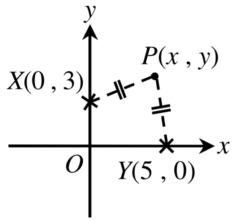
$\therefore$ The equation of the locus of P is $2x + y + 3 = 0$.

The locus of P is the perpendicular bisector of MN.



Instant Practice 2

In the figure, $X(0 , 3)$ and $Y(5 , 0)$ are two points in a rectangular coordinate plane. A point $P(x , y)$ moves such that $PX = PY$. Find the equation of the locus of P.



$\because \quad (\quad\quad) = (\quad\quad)$

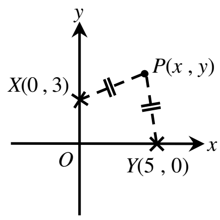
$\therefore \quad \sqrt{[x-(\quad)]^2+[y-(\quad)]^2}$

$= \sqrt{[x-(\quad)]^2+[y-(\quad)]^2}$

$$x^2 + [y - (\quad)]^2 = [x - (\quad)]^2 + y^2$$
$$x^2 + y^2 - (\quad)y + (\quad) = x^2 - (\quad)x + (\quad) + y^2$$
$$(\quad)x - (\quad)y - (\quad) = 0$$
$$(\quad)x - (\quad)y - (\quad) = 0$$

$\therefore$ The equation of the locus of P is ____________________.

The locus of P is ____ ________________ ________________.



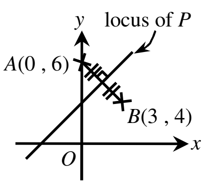
3. $A(0 , 6)$ and $B(3 , 4)$ are two points in a rectangular coordinate plane. A moving point $P(x , y)$ is equidistant from A and B. Find the equation of the locus of P.

$\because \quad (\quad\quad) = (\quad\quad)$

$\therefore \quad \sqrt{[x-(\quad)]^2+[y-(\quad)]^2}$

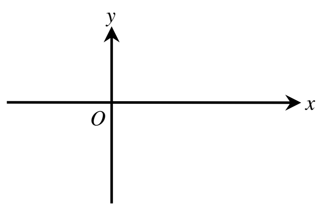
$= \sqrt{[x-(\quad)]^2+[y-(\quad)]^2}$

In this question, no figure is given. Make a sketch and check if the answer obtained has the correct slope and y-intercept.



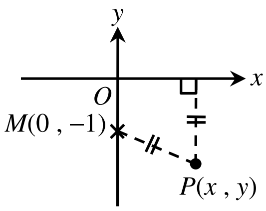
4. $R(7 , -4)$ and $S(8 , 1)$ are two points in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from R and S. Find the equation of the locus of P. ➥Ex 8B: 5–9

Make a sketch:



Instant Example 3

In the figure, a moving point $P(x\ ,\ y)$ maintains an equal distance from $M(0\ ,\ -1)$ and the x-axis. Find the equation of the locus of P.



Let Q be a point on the x-axis such that PQ is perpendicular to the x-axis.

$PQ = 0 - y = -y$ ◄ P is below the x-axis.

$PM = \sqrt{(x-0)^2 + [y-(-1)]^2}$

$= \sqrt{x^2 + (y+1)^2}$

$\because\ PM = PQ$

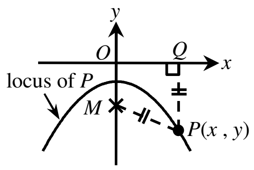
$\therefore\ \sqrt{x^2+(y+1)^2} = -y$

$x^2 + (y+1)^2 = (-y)^2$

$x^2 + y^2 + 2y + 1 = y^2$

$x^2 + 2y + 1 = 0$

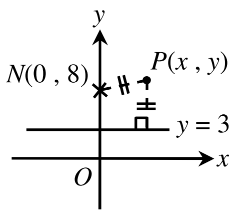
$\therefore$ The equation of the locus of P is $x^2 + 2y + 1 = 0$.



The locus of P is a parabola which opens downward.

Instant Practice 3

In the figure, a moving point $P(x\ ,\ y)$ maintains an equal distance from $N(0\ ,\ 8)$ and the line $y = 3$. Find the equation of the locus of P.



Let Q be a point on the line ( ) such that PQ is ( ) the line ( ).

$PQ = (\quad) - (\quad)$

$PN = \sqrt{[x-(\quad)]^2 + [y-(\quad)]^2}$

$= \sqrt{(\quad)^2 + (\quad)^2}$

$\because\ PN = (\quad)$

$\therefore\ \sqrt{(\quad)^2 + (\quad)^2} = (\quad)$

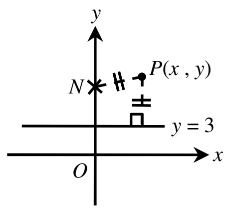
$(\quad)^2 + (\quad)^2 = (\quad)^2$

$x^2 + y^2 - (\quad)y + (\quad) = y^2 - (\quad)y + (\quad)$

$x^2 - (\quad)y + (\quad) = 0$

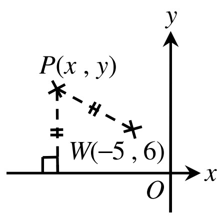
$\therefore$ The equation of the locus of P is ______________________.

Sketch the locus of P.



The locus of P is a ( ) which opens ( ).

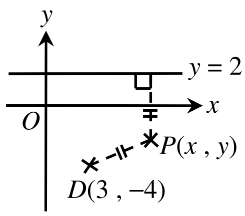
5. In the figure, a moving point $P(x\ ,\ y)$ maintains an equal distance from $W(-5\ ,\ 6)$ and the x-axis. Find the equation of the locus of P.



Let Q be a point on the ( ) such that PQ is ( ) the ( ).

$PQ = (\quad) - (\quad) = (\quad)$

6. In the figure, a moving point $P(x\ ,\ y)$ maintains an equal distance from $D(3\ ,\ -4)$ and the line L: $y = 2$. Find the equation of the locus of P.



➥Ex 8B: 11–14, 17, 18

7. $A(-3, -9)$ is a point in a rectangular coordinate plane. A moving point $P(x, y)$ maintains a fixed distance from A. The fixed distance is equal to the distance between A and the y-axis.

(a) Find the equation of the locus of P.

(b) A moving point $Q(x, y)$ maintains a fixed distance from A. The fixed distance is larger than the distance between P and A by 2. Find the equation of the locus of Q.

8. $L: 3x + 2y - 6 = 0$ is a straight line in a rectangular coordinate plane. P is a moving point such that it maintains a fixed distance from L and P lies above L. Denote the locus of P by Γ.

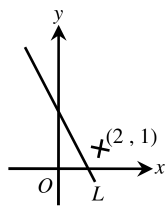
(a) Describe the geometric relationship between L and Γ.

(b) If Γ passes through $(2, 1)$, find the equation of Γ.

(a) Γ is ( ) L.

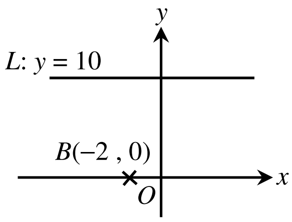
(b) Slope of Γ = slope of ( )

Sketch the locus of Γ.



⇧Level Up Question⇧

9. In the figure, $L: y = 10$ is a straight line in a rectangular coordinate plane.



(a) A moving point $P(x, y)$ maintains an equal distance from L and the x-axis. Find the equation of the locus of P.

(b) A moving point $Q(x, y)$ maintains an equal distance from $B(-2, 0)$ and L. Find the equation of the locus of Q.

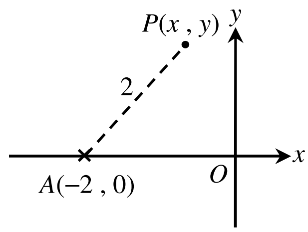
Explain **(c)** Does the locus of P intersect the locus of Q? Explain your answer.

8 Locus

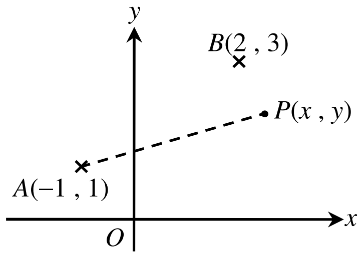
Consolidation Exercise 8B

Level 1

1. In the figure, $A(-2, 0)$ is a point on the x-axis. A moving point $P(x, y)$ maintains a fixed distance of 2 from A. Find the equation of the locus of P.



2. In the figure, $A(-1, 1)$ and $B(2, 3)$ are two points in a rectangular coordinate plane. A point $P(x, y)$ moves such that $PA = AB$. Find the equation of the locus of P.



In each of the following, A is a point in a rectangular coordinate plane. A moving point $P(x, y)$ maintains a fixed distance of d from A. Find the equation of the locus of P. **[Nos. 3–4]**

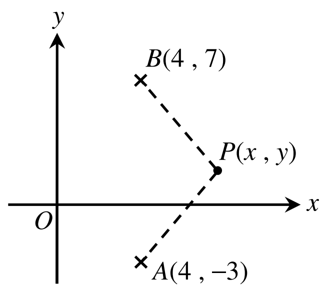
3. $A(12, 9)$ and $d = 14$

4. $A(7, -4)$ and $d = \sqrt{21}$

5. In the figure, $A(4, -3)$ and $B(4, 7)$ are two points in a rectangular coordinate plane. A moving point $P(x, y)$ maintains an equal distance from A and B, i.e. $PA = PB$.

(a) Find the equation of the locus of P.

(b) Describe the geometric relationship between the locus of P and the line $y = 1$.



6. $C(-5, 6)$ and $D(-2, 6)$ are two points in a rectangular coordinate plane. A moving point $P(x, y)$ maintains an equal distance from C and D, i.e. $PC = PD$.

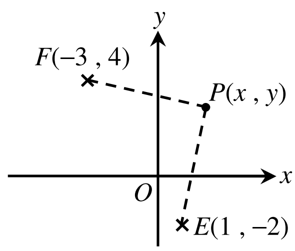
(a) Find the equation of the locus of P.

(b) Describe the geometric relationship between the locus of P and the x-axis.

7. In the figure, $E(1, -2)$ and $F(-3, 4)$ are two points in a rectangular coordinate plane. A moving point $P(x, y)$ maintains an equal distance from E and F.

(a) Describe the locus of P.

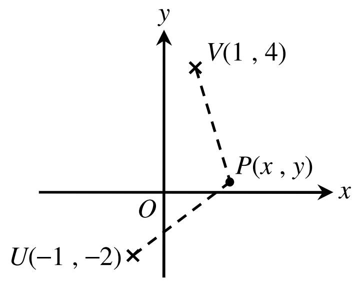
(b) Find the equation of the locus of P.



8. In the figure, U and V are two points in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from U and V.

(a) Find the equation of the locus of P.

(b) The equation of a straight line L is $y = 3x + 2$. Describe the geometric relationship between the locus of P and L.



In each of the following, A and B are two points in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from A and B. Find the equation of the locus of P. **[Nos. 9–10]**

9. $A(1 , 1)$ and $B(0 , 6)$

10. $A(-2 , -5)$ and $B(1 , -4)$

$A(3 , 2)$ and $B(6 , 6)$ are two points in a rectangular coordinate plane. Find the equation of the locus of a moving point $P(x , y)$ satisfying each of the following conditions. **[Nos. 11–12]**

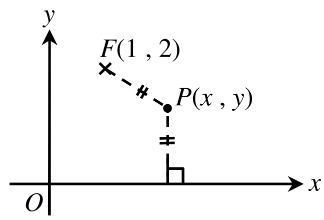
11. $3AP = PB$

12. $AP : PB = 2 : 1$

13. In the figure, $F(1 , 2)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from F and the x-axis.

(a) Describe the locus of P.

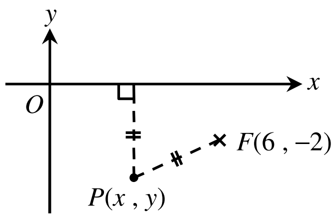
(b) Find the equation of the locus of P.



14. In the figure, $F(6 , -2)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from F and the x-axis.

(a) Find the equation of the locus of P.

(b) Find the y-intercept of the locus of P.



In each of the following, F is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from F and the x-axis. Find the equation of the locus of P. **[Nos. 15–16]**

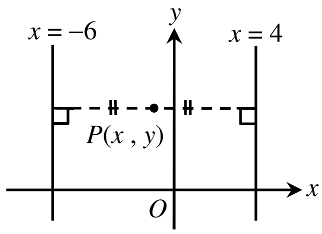
15. $F(-7 , 7)$

16. $F(-4 , -3)$

17. In the figure, a moving point $P(x , y)$ maintains an equal distance from the vertical line $x = 4$ and $x = -6$.

(a) Find the equation of the locus of P.

Explain **(b)** Is $(-2 , 5)$ a point on the locus of P? Explain your answer.



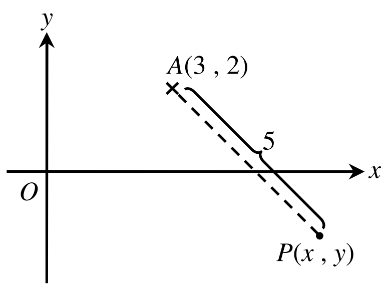
Level 2

18. In the figure, $A(3 , 2)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains a fixed distance of 5 from A.

(a) Find the equation of the locus of P.

(b) If $Q(x , y)$ is a point on the line segment AP such that $AQ : QP = 3 : 2$, find the equation of the locus of Q.

(c) Describe the geometric relationship between the loci of P and Q.



19. $F(-5\ ,\ 2)$ is a point in a rectangular coordinate plane. A moving point $P(x\ ,\ y)$ maintains an equal distance from F and the line $y = -2$.

(a) Find the equation of the locus of P.

Explain **(b)** Does the locus of P pass through $(4\ ,\ 10)$? Explain your answer.

20. $F(4\ ,\ 1)$ is a point in a rectangular coordinate plane. A moving point $P(x\ ,\ y)$ maintains an equal distance from F and the line $y = 4$.

(a) Find the equation of the locus of P.

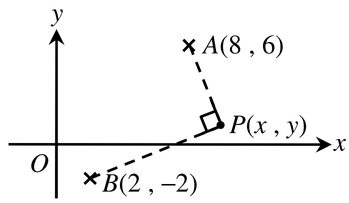
(b) Using the method of completing the square, find the coordinates of the vertex of the locus of P.

21. $F(6\ ,\ 3)$ is a point in a rectangular coordinate plane. A point $P(x\ ,\ y)$ maintains an equal distance from the point F and the line $y = 5$. The locus of P intersects the x-axis at two points A and B.

(a) Find the equation of the locus of P.

Explain **(b)** If R is a point on the locus of P and R lies above the x-axis, can the area of $\triangle RAB$ exceed 16? Explain your answer.

22. In the figure, $A(8\ ,\ 6)$ and $B(2\ ,\ -2)$ are two points in a rectangular coordinate plane. A point $P(x\ ,\ y)$ moves such that $\angle APB = 90°$.



(a) Find the equation of the locus of P.

(b) Describe the locus of P.

23. $A(7\ ,\ -1)$ and $B(4\ ,\ 5)$ are two points in a rectangular coordinate plane. A point $P(x\ ,\ y)$ moves such that $PA \perp PB$.

(a) Find the equation of the locus of P.

Explain **(b)** Does $(6\ ,\ -1)$ lie on the locus of P? Explain your answer.

24. $A(3\ ,\ 7)$ and $B(-5\ ,\ 1)$ are two points in a rectangular coordinate plane. A point $P(x\ ,\ y)$ moves such that $AP^2 + PB^2 = AB^2$.

(a) Find the equation of the locus of P.

(b) If a point Q lies on the locus of P and the x-coordinate of Q is -1, find the possible coordinates of Q.

25. In a rectangular coordinate plane, a moving point $P(x\ ,\ y)$ maintains a fixed distance of 4 from the line $y = -3$ and P lies above the line $y = -3$.

(a) Find the equation of the locus of P.

(b) $A(3\ ,\ -4)$ is reflected to A' with the locus of P as the axis of reflection. A' is then rotated clockwise about the origin O through 90° to B. Find the coordinates of B.

26. $A(-5\ ,\ 13)$ and $B(-3\ ,\ -1)$ are the end points of a diameter of a circle C.

(a) Find the equation of C in the standard form.

(b) $P(x\ ,\ y)$ is a moving point in the rectangular coordinate plane such that $AP = PB$.

(i) Find the equation of the locus of P.

(ii) If the locus of P cuts C at Q and R, find the perimeter of $\triangle AQR$.

(*Leave the radical sign '$\sqrt{\ }$' in the answer.*)

27. The line L: $4x + 3y - 12 = 0$ cuts the x-axis and the y-axis at the points A and B respectively. $C(c , 0)$ is a point on the x-axis such that the area of $\triangle ABC$ is twice the area of $\triangle OAB$.

(a) Find the values of c.

(b) Take c as the larger value obtained in **(a)**. $P(x , y)$ is a moving point on the right of L in the rectangular coordinate plane such that the area of $\triangle PAB$ is equal to that of $\triangle ABC$.

(i) Describe the geometric relationship between the locus of P and L.

(ii) Find the equation of the locus of P.

✂ * **28.** $A(0 , 2)$, $B(14 , 10)$ and $C(16 , 0)$ are three points in a rectangular coordinate plane. G is the centroid of $\triangle ABC$.

(a) Find the coordinates of G.

(b) A point $P(x , y)$ moves such that $PG = PC$.

(i) Find the equation of the locus of P.

(ii) Describe the geometric relationship between the locus of P and the line segment BG.

Explain **(iii)** Does the height of $\triangle PBG$ with BG as the base change when P moves? Explain your answer.

* **29.** $A(-1 , -3)$ is rotated anticlockwise about the origin O through 90° to A'. B' is the image when $B(-9 , -1)$ is reflected in the y-axis. A point $P(x , y)$ moves such that $A'P \perp PB'$.

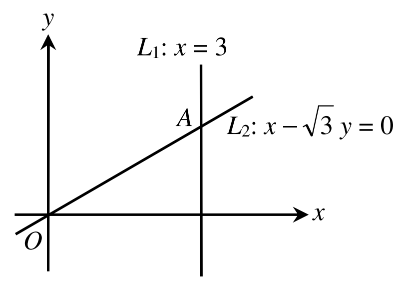
(a) Write down the coordinates of A' and B'.

(b) Find the equation of the locus of P.

✂ **(c)** Find the coordinates of the circumcentre of $\triangle A'PB'$.

✂ **(d)** Let Q be the centroid of $\triangle A'PB'$. When P moves, find the equation of the locus of Q.

✂ * **30.** In the figure, two lines L_1: $x = 3$ and L_2: $x - \sqrt{3}\,y = 0$ intersect at a point A. A point $P(x , y)$ lies below L_2 and on the left of L_1, where $x < 3$. When P moves, it maintains an equal distance from L_1 and L_2. Denote the locus of P by Γ.



(a) Find the inclination of Γ. Hence, find the equation of Γ, where $x < 3$.

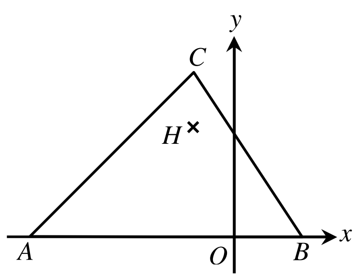
(b) C is a circle with centre G lying below the x-axis. L_1 touches C and L_2 is the tangent to C at $Q(-3 , -\sqrt{3})$.

(i) Find the equation of C.

(ii) If Γ intersects the x-axis at R, find the ratio of the area of $\triangle QGR$ to that of $\triangle QRA$.

(*Leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*)

✂ * **31.** In the figure, $A(-15 , 0)$, $B(5 , 0)$ and $C(-3 , 12)$ are three points in a rectangular coordinate plane. H is the orthocentre of $\triangle ABC$.



(a) Find the coordinates of H.

(b) A moving point $P(x , y)$ maintains an equal distance from B and H.

(i) Find the equation of the locus of P.

Explain **(ii)** Do the locus of P and the straight line passing through A and C intersect? Explain your answer.

(iii) Suppose the locus of P intersects BC and the x-axis at D and E respectively. Find the ratio of the area of $\triangle BDE$ to that of the quadrilateral $DEAC$.

Answers

Consolidation Exercise 8B

1. $(x + 2)^2 + y^2 = 4$

2. $(x + 1)^2 + (y - 1)^2 = 13$

3. $(x - 12)^2 + (y - 9)^2 = 196$

4. $(x - 7)^2 + (y + 4)^2 = 21$

5. **(a)** $y = 2$

(b) The locus of P is parallel to the line $y = 1$.

6. **(a)** $x = -\frac{7}{2}$

(b) The locus of P is perpendicular to the x-axis.

7. **(a)** the perpendicular bisector of EF

(b) $2x - 3y + 5 = 0$

8. **(a)** $x + 3y - 3 = 0$

(b) The locus of P is perpendicular to L.

9. $x - 5y + 17 = 0$

10. $3x + y + 6 = 0$

11. $8x^2 + 8y^2 - 42x - 24y + 45 = 0$

12. $3x^2 + 3y^2 - 42x - 44y + 275 = 0$

13. **(a)** The locus of P is a parabola opening upward.

(b) $x^2 - 2x - 4y + 5 = 0$

14. **(a)** $x^2 - 12x + 4y + 40 = 0$

(b) -10

15. $x^2 + 14x - 14y + 98 = 0$

16. $x^2 + 8x + 6y + 25 = 0$

17. **(a)** $x = -1$ **(b)** no

18. **(a)** $(x - 3)^2 + (y - 2)^2 = 25$

(b) $(x - 3)^2 + (y - 2)^2 = 9$

(c) The loci of P and Q are concentric circles.

19. **(a)** $x^2 + 10x - 8y + 25 = 0$

(b) no

20. **(a)** $x^2 - 8x + 6y + 1 = 0$

(b) $\left(4, \frac{5}{2}\right)$

21. **(a)** $x^2 - 12x + 4y + 20 = 0$ **(b)** no

22. **(a)** $x^2 + y^2 - 10x - 4y + 4 = 0$, excluding points $A(8, 6)$ and $B(2, -2)$

(b) The locus of P is a circle with centre $(5, 2)$ and radius 5, excluding points A and B.

23. **(a)** $x^2 + y^2 - 11x - 4y + 23 = 0$, excluding points $A(7, -1)$ and $B(4, 5)$

(b) no

24. **(a)** $x^2 + y^2 + 2x - 8y - 8 = 0$

(b) $(-1, -1)$, $(-1, 9)$

25. **(a)** $y = 1$

(b) $(6, -3)$

26. **(a)** $(x + 4)^2 + (y - 6)^2 = 50$

(b) $x - 7y + 46 = 0$

(c) $20 + 2\sqrt{50}$ (or $20 + 10\sqrt{2}$)

27. **(a)** -3, 9

(b) **(i)** The locus of P is a straight line parallel to L and passing through C.

(ii) $4x + 3y - 36 = 0$

28. **(a)** $(10, 4)$

(b) **(i)** $3x - 2y - 35 = 0$

(ii) The locus of P is parallel to the line segment BG.

(iii) no

29. **(a)** A': $(3, -1)$, B': $(9, -1)$

(b) $x^2 + y^2 - 12x + 2y + 28 = 0$, excluding points $A'(3, -1)$ and $B'(9, -1)$

(c) $(6, -1)$

(d) $(x - 6)^2 + (y + 1)^2 = 1$, excluding points $(5, -1)$ and $(7, -1)$

30. **(a)** inclination of Γ: 60°, equation of Γ: $\sqrt{3}x - y - 2\sqrt{3} = 0$

(b) **(i)** $(x + 1)^2 + (y + 3\sqrt{3})^2 = 16$

(ii) 3 : 1

31. **(a)** $(-3, 8)$

(b) **(i)** $x - y + 3 = 0$ **(ii)** no

(iii) 4 : 2

F5B: Chapter 9A

Date	Task	Progress	
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)
	Book Example 1	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 2	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 3	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 4	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 5	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 6	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)
	Maths Corner Exercise 9A Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher’s Signature ________ ( )

	Maths Corner Exercise 9A Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 9A Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: _________	

5B Lesson Worksheet 9.0

(Refer to Book 5B P.9.3)

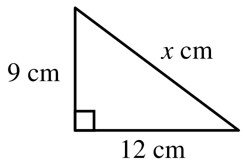
Objective: To review Pythagoras' theorem, trigonometric ratios and trigonometric equations.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

Pythagoras' Theorem and Trigonometric Ratios

In each of the following, find the value of x. **[Nos. 1–2]**

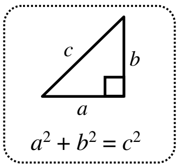
1.



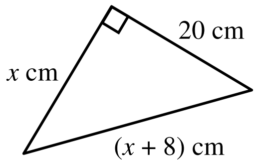
$(\quad)^2 + (\quad)^2 = (\quad)^2$

$x^2 = (\quad)$

$x = \underline{\underline{\qquad}}$



2.



In each of the following, find the values of x and y. **[Nos. 3–4]**

➥Review Ex: 1

3. $\tan 32° = \dfrac{(\quad)}{(\quad)}$

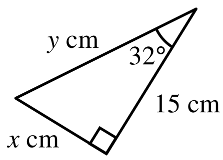
$x = (\quad)(\qquad)$

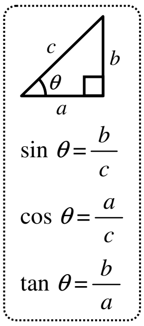
$= \underline{\underline{\qquad}}$, *cor. to 3 sig. fig.*

$\cos 32° = \dfrac{(\quad)}{(\quad)}$

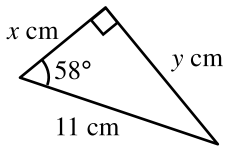
$y = \dfrac{(\quad)}{(\qquad)}$

$= \underline{\underline{\qquad}}$, *cor. to 3 sig. fig.*





4.



In each of the following, find the values of sin θ, cos θ and tan θ. **[Nos. 5–6]**

➥Review Ex: 2

5. $AC^2 = (\quad)^2 + (\quad)^2$

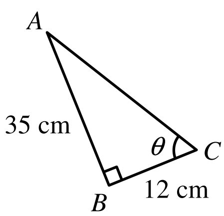
$AC = \sqrt{(\quad)^2 + (\quad)^2}$ cm

$= (\quad)$ cm

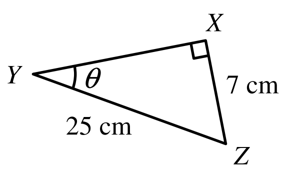
$\sin\theta = \dfrac{(\quad)}{AC} = \underline{\underline{\dfrac{(\quad)}{(\quad)}}}$

$\cos\theta = \dfrac{(\quad)}{(\quad)} = \underline{\underline{\qquad}}$

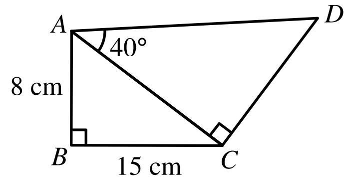
$\tan\theta = \dfrac{(\quad)}{(\quad)} = \underline{\underline{\qquad}}$



6.



7. In the figure, $AB = 8$ cm, $BC = 15$ cm and $\angle DAC = 40°$. $\angle ABC$ and $\angle ACD$ are right angles. Find the area of $\triangle ACD$.



Trigonometric Equations

Solve the following equations, where $0° < \theta < 180°$. **[Nos. 8–11]**

8. $\sin \theta = 0.8$

$\theta = ($ $\quad\quad\quad$ $)$ ← SHIFT sin 0.8 EXE

or ($\quad\quad\quad\quad\quad$)

= ______ *or* ______, *cor. to 3 sig. fig.*

∵ $\sin \theta$ (< / >) 0
∴ θ lies in quadrant ( ) or quadrant ( ).

9. $\tan \theta = -0.7$

➥Review Ex: 3

Note that $0° < \theta < 180°$.

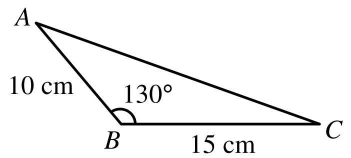
SHIFT tan 0.7 EXE

10. $3 \cos \theta = -2$

11. $4 \tan \theta = 9$

⇪Level Up Question⇪

12. In the figure, $AB = 10$ cm, $BC = 15$ cm and $\angle ABC = 130°$.
Find the area of $\triangle ABC$.



When BC is the base, what is the height of $\triangle ABC$?

✂ 5B Lesson Worksheet 9.1A

(Refer to Book 5B P.9.4)

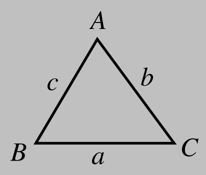
Objective: To use the sine formula to find unknown side/angle when two angles and one side are given.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

Sine Formula

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Denote ∠A, ∠B and ∠C by A, B and C respectively. Denote the lengths of the sides opposite to ∠A, ∠B and ∠C by a, b and c respectively.



Given Two Angles and One Side (*AAS* or *ASA*)

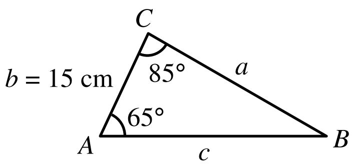
If two angles and one side of a triangle are given, the triangle can be solved by the following steps.

(i) Use the angle sum of triangle to find the remaining angle.

(ii) Use the sine formula to find the other two sides.

Instant Example 1

In △ ABC, A = 65°, C = 85° and b = 15 cm. Solve the triangle.



$A + B + C = 180°$ (*∠ sum of △*)

$65° + B + 85° = 180°$

$B = \underline{\underline{30°}}$ ◄ **Step (i)**

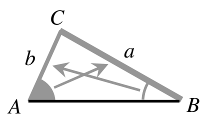
By the sine formula, ◄ **Step (ii)**

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

$$\frac{a}{\sin 65°}=\frac{15\text{ cm}}{\sin 30°}$$

$$a=\frac{15\sin 65°}{\sin 30°}\text{ cm}$$

$= \underline{\underline{27.2\text{ cm}}}$, *cor. to 3 sig. fig.*

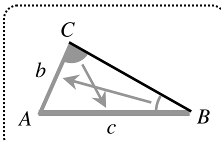


$$\frac{c}{\sin C}=\frac{b}{\sin B}$$

$$\frac{c}{\sin 85°}=\frac{15\text{ cm}}{\sin 30°}$$

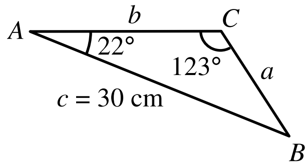
$$c=\frac{15\sin 85°}{\sin 30°}\text{ cm}$$

$= \underline{\underline{29.9\text{ cm}}}$, *cor. to 3 sig. fig.*



Instant Practice 1

In △ ABC, A = 22°, C = 123° and c = 30 cm. Solve the triangle.



$A + B + C = 180°$

$(\quad\quad) + B + (\quad\quad) = 180°$

$B =$ ______

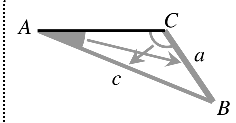
By the sine formula,

$$\frac{a}{\sin A}=\frac{(\quad)}{\sin(\quad)}$$

$$\frac{a}{\sin(\quad\quad)}=\frac{(\quad\quad)}{\sin(\quad\quad)}$$

$$a=\frac{(\quad)(\quad\quad\quad)}{(\quad\quad\quad)}\text{ cm}$$

$=$ ________, *cor. to 3 sig. fig.*



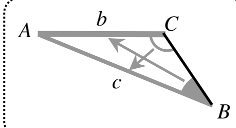
∠C and its opposite side c are known.

$$\frac{b}{\sin B}=\frac{(\quad)}{\sin C}$$

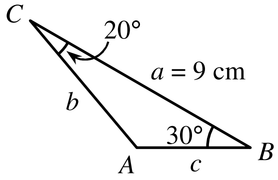
$$\frac{b}{\sin(\quad\quad)}=\frac{(\quad\quad)}{\sin(\quad\quad)}$$

$$b=\frac{(\quad)(\quad\quad\quad)}{(\quad\quad\quad)}\text{ cm}$$

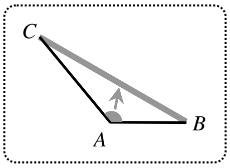
$=$ ________, *cor. to 3 sig. fig.*



1. In $\triangle ABC$, $B = 30°$, $C = 20°$ and $a = 9$ cm. Solve the triangle.

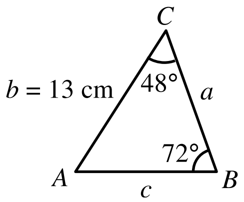


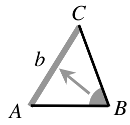
$A + B + C = 180°$



2. In $\triangle ABC$, $B = 72°$, $C = 48°$ and $b = 13$ cm. Solve the triangle.

➥Ex 9A: 7–9





∠B and its opposite side b are known.

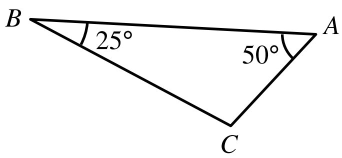
⇧Level Up Question⇧

3. In $\triangle ABC$, $A = 50°$ and $B = 25°$.

Explain

(a) Is it true that $BC = 2AC$? Explain your answer.

(b) If $BC = 10$ cm, is it possible that $AB > 13$ cm? Explain your answer.



✂ 5B Lesson Worksheet 9.1B

(Refer to Book 5B P.9.7)

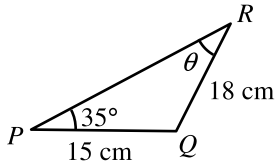
Objective: To use the sine formula to find unknown side/angle when two sides and one opposite angle are given.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

Sine Formula: Given Two Sides and One Opposite Angle (*SSA*)

Instant Example 1

In the figure, find θ.



By the sine formula,

$$\frac{15\text{ cm}}{\sin\theta} = \frac{18\text{ cm}}{\sin 35°}$$

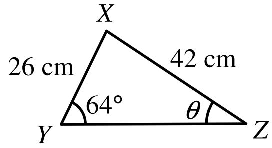
$$\sin\theta = \frac{15\sin 35°}{18}$$

$$\theta = \underline{\underline{28.6°}},\ \textit{cor. to 3 sig. fig.}$$

$\because$ $PQ < QR$
$\therefore$ θ must be an acute angle smaller than 35°.

Instant Practice 1

In the figure, find θ.



By the sine formula,

$$\frac{(\qquad)}{\sin\theta} = \frac{(\qquad)}{\sin(\qquad)}$$

$$\sin\theta = \frac{(\quad)\sin(\quad)}{(\quad)}$$

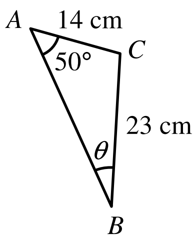
$\theta =$ ________, *cor. to 3 sig. fig.*

In each of the following triangles, find θ. **[Nos. 1–4]**

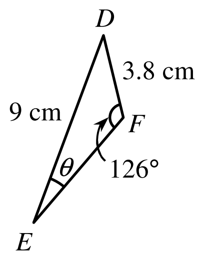
➥Ex 9A: 10–15

1. By the sine formula,

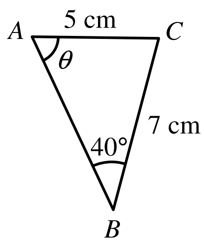
$$\frac{(\qquad)}{\sin\theta} = \frac{(\qquad)}{\sin(\qquad)}$$



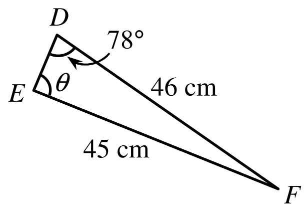
2.



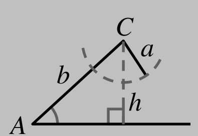
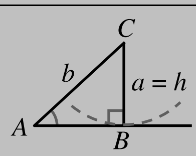
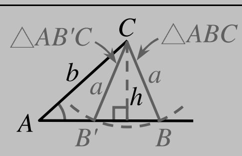
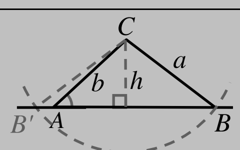
3. θ is acute.



4. θ is obtuse.



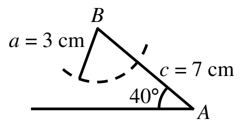
When two sides a, b and one **acute** opposite angle A are given, the number of triangles that can be formed is 0, 1 or 2. In each figure below, let h be the perpendicular distance from the vertex C to its opposite side, where $h = b\sin A$.

Case 1: $a < h$	**Case 2:** $a = h$	**Case 3:** $h < a < b$	**Case 4:** $a \geq b$
			
No triangles can be formed.	One triangle is formed.	Two triangles are formed.	One triangle is formed.

Instant Example 2

Determine whether $\triangle ABC$ can be formed if $A = 40°$, $c = 7$ cm and $a = 3$ cm.

By the sine formula,



$$\frac{7\text{ cm}}{\sin C} = \frac{3\text{ cm}}{\sin 40°}$$

$$\sin C = \frac{7\sin 40°}{3}$$

$= 1.50$, *cor. to 3 sig. fig.*

> 1

$\therefore$ There is no solution for C. ◄ $-1 \le \sin C \le 1$ for all values of C.

$\therefore$ <u>$\triangle ABC$ cannot be formed.</u>

Instant Practice 2

Determine whether $\triangle ABC$ can be formed if $A = 45°$, $c = 5\sqrt{2}$ cm and $a = 5$ cm.

By the sine formula,

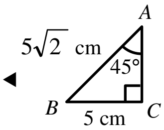
$$\frac{(\qquad\qquad)}{\sin C} = \frac{(\qquad)}{\sin(\qquad)}$$

$$\sin C = \frac{(\qquad)\sin(\qquad)}{(\qquad)}$$

$= (\qquad)$

$C = (\qquad)$

$\therefore$ <u>$\triangle ABC$ (can / cannot) be formed.</u> ◄



In each of the following, determine whether $\triangle ABC$ can be formed. **[Nos. 5–6]** ➥Ex 9A: 16, 17

5. $B = 30°$, $a = 4$ cm, $b = 2$ cm

By the sine formula,

$$\frac{(\qquad)}{\sin A} = \frac{(\qquad)}{\sin(\qquad)}$$

6. $C = 70°$, $b = 12$ cm, $c = 10$ cm

For any θ, $-1 \le \sin\theta \le 1$.

⇪Level Up Question⇪

7. In $\triangle ABC$, $B = 30°$ and $AB = 20$ cm.

Explain **(a)** Kelvin claims that the minimum length of AC is 10 cm. Do you agree? Explain your answer.

(b) Find the range of the length of AC such that two triangles can be formed.

9 Solving Triangles

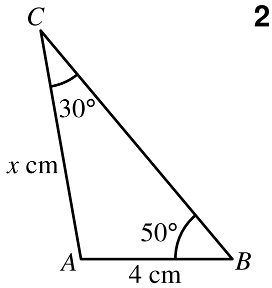
✂ Consolidation Exercise 9A

[*In this exercise, unless otherwise stated, give the answers correct to 3 significant figures if necessary.*]

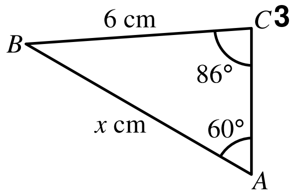
Level 1

In each of the following triangles, find the value of x. **[Nos. 1–6]**

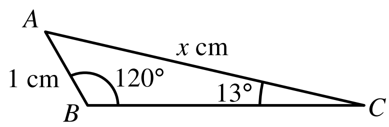
1.



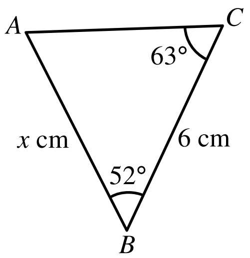
2.



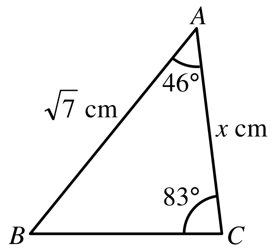
3.



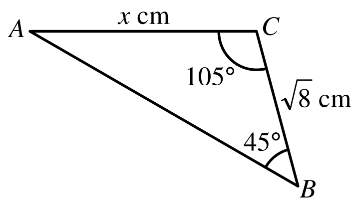
4.



5.

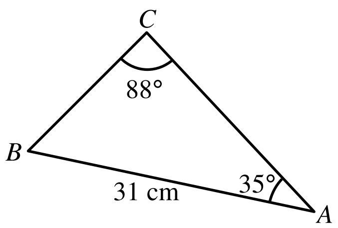


6.

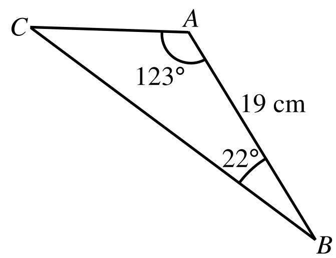


Solve the following triangles. **[Nos. 7–9]**

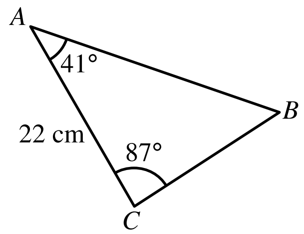
7.



8.

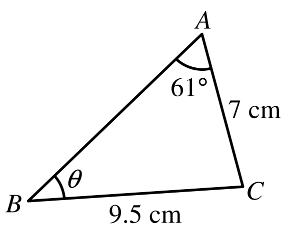


9.

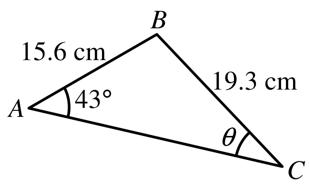


In each of the following triangles, find θ. **[Nos. 10–15]**

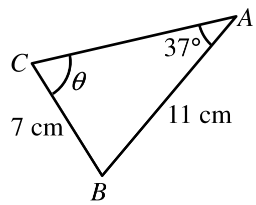
10.



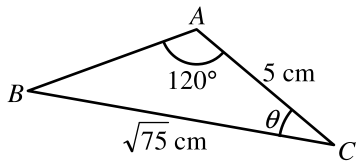
11.



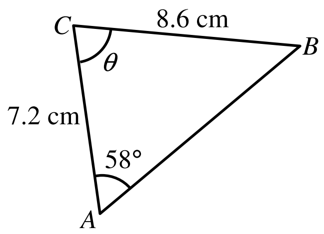
12. θ is acute.



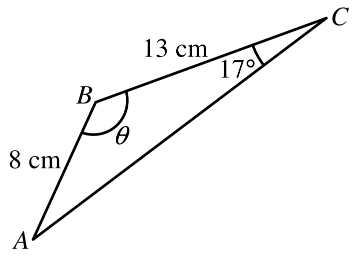
13.



14.



15. θ is obtuse.



In each of the following, determine whether $\triangle ABC$ can be formed. If yes, find C. **[Nos. 16–19]**

16. $A = 25°$, $a = 11$ cm, $c = 27$ cm

17. $A = 74°$, $a = 16$ cm, $c = 9$ cm

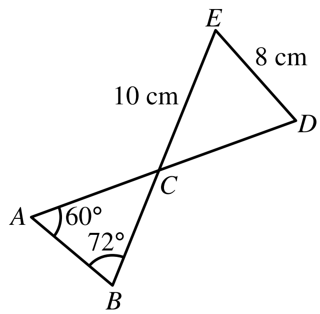
18. $B = 135°$, $c = 7$ cm, $b = 2$ cm

19. $B = 103°$, $a = 8$ cm, $b = 3\sqrt{5}$ cm

20. In the figure, ACD and BCE are two straight lines. $CE = 10$ cm, $DE = 8$ cm, $\angle ABC = 72°$ and $\angle BAC = 60°$.

(a) Find $\angle DCE$.

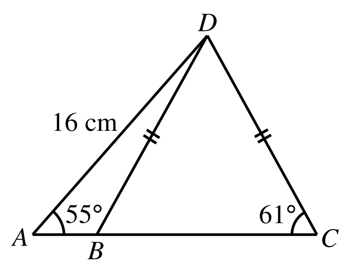
(b) If $\angle CDE$ is an acute angle, find $\angle CDE$.



21. In the figure, ABC is a straight line and $BD = CD$. $AD = 16$ cm, $\angle BAD = 55°$ and $\angle BCD = 61°$.

(a) Find $\angle ADB$.

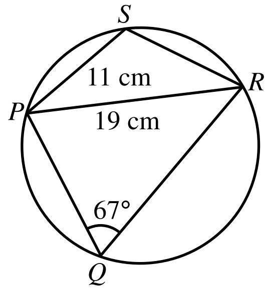
(b) Find the length of AB.



22. In the figure, P, Q, R and S are points on a circle. $PR = 19$ cm, $PS = 11$ cm and $\angle PQR = 67°$.

(a) Find $\angle SPR$.

Explain **(b)** Is $\triangle PRS$ an isosceles triangle? Explain your answer.



Level 2

23. In each of the following, find a in $\triangle ABC$.

(a) $B = 65°$, $C = 20°$, $b = 7$ cm

(b) $B = 57°$, $C = 43°$, $c = 18.5$ cm

24. In each of the following, find B in $\triangle ABC$.

(a) $C = 62°$, $b = 18$ cm, $c = 17$ cm

(b) $A = 47°$, $a = 11$ cm, $b = 15$ cm

(c) $C = 124°$, $b = 4.2$ cm, $c = 8.6$ cm

In each of the following, determine whether a triangle can be formed. If yes, solve the triangle. **[Nos. 25–26]**

25. (a) In $\triangle ABC$, $A = 31°$, $C = 13°$ and $c = 11$ cm.

(b) In $\triangle LMN$, $L = 59°$, $M = 35°$ and $n = 4.5$ cm.

(c) In $\triangle DEF$, $E = 101°$, $F = 29°$ and $d = 17.5$ cm.

26. (a) In $\triangle ABC$, $B = 47°$, $b = 4$ cm and $c = 14$ cm.

(b) In $\triangle PQR$, $Q = 99°$, $p = 17$ cm and $q = 19$ cm.

(c) In $\triangle RST$, $T = 21°$, $s = 8.5$ cm and $t = 4.1$ cm.

(d) In $\triangle XYZ$, $Y = 51°$, $x = 6.5$ cm and $y = 5.5$ cm.

27. It is given that $DE = 21$ cm and $\angle DEF = 30°$. Find the range of the length of DF such that

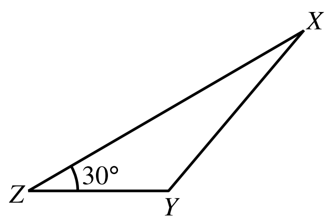
(a) only one triangle can be formed,

(b) two triangles can be formed.

28. In the figure, $Z = 30°$ and $YZ : XY = 2 : 3$.

(a) Find X.

(b) Find the value of $\dfrac{XZ}{XY}$.

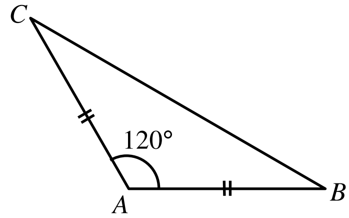


29. In the figure, $AB = AC$ and $A = 120°$.

(a) Find $BC : AC : AB$.

(*Leave the radical sign '$\sqrt{\ }$' in the answer.*)

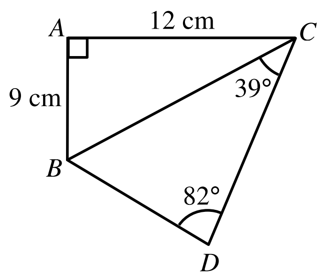
(b) If $BC = 9$ cm, find the perimeter of $\triangle ABC$.



30. In the figure, $AB = 9$ cm, $AC = 12$ cm, $\angle BCD = 39°$, $\angle CDB = 82°$ and $\angle BAC$ is a right angle.

(a) Find the length of BD.

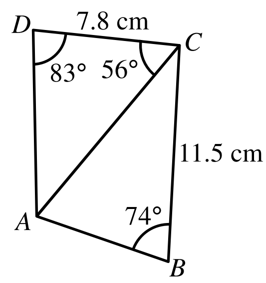
(b) Find the perimeter of the quadrilateral $ABDC$.



31. In the figure, $BC = 11.5$ cm, $CD = 7.8$ cm, $\angle ABC = 74°$, $\angle ACD = 56°$ and $\angle ADC = 83°$.

(a) Find $\angle BAC$.

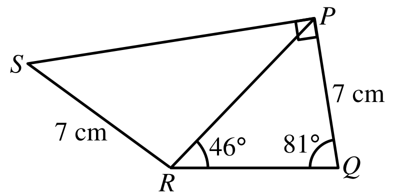
(b) Find the perimeter of $\triangle ABC$.



32. In the figure, $PQ = RS = 7$ cm, $\angle PQR = 81°$ and $\angle PRQ = 46°$. $\angle QPS$ is a right angle and $\angle PSR$ is an acute angle.

(a) Find the length of PR.

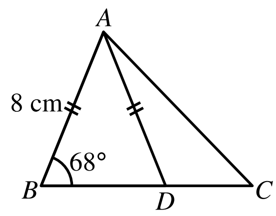
(b) Find the length of PS.



33. In the figure, BDC is a straight line. $\angle BAD = 2\angle DAC$, $\angle ABD = 68°$ and $AB = AD = 8$ cm.

(a) Find the length of BC.

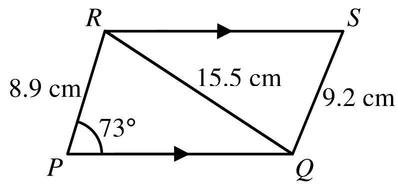
Explain **(b)** Which triangle, $\triangle ABD$ or $\triangle ADC$, has a greater perimeter? Explain your answer.



34. In the figure, PQ // RS, $PR = 8.9$ cm, $QR = 15.5$ cm, $QS = 9.2$ cm and $\angle QPR = 73°$. RS is the longest side in $\triangle QRS$.

(a) Find $\angle QRS$.

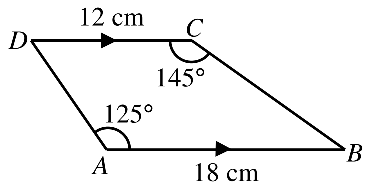
(b) Find the length of RS.



35. In the figure, AB // DC, $AB = 18$ cm, $DC = 12$ cm, $\angle DAB = 125°$ and $\angle DCB = 145°$.

(a) Find the length of BC.

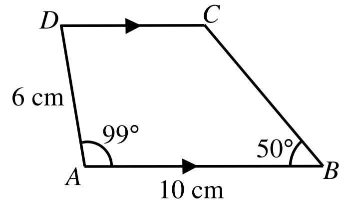
(b) Find the length of AD.



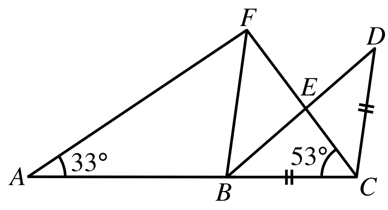
36. The figure shows a trapezium $ABCD$, where AB // DC. $AB = 10$ cm, $AD = 6$ cm, $\angle ABC = 50°$ and $\angle BAD = 99°$.

(a) Find the lengths of BC and CD.

(b) Find the area of $ABCD$.



37. In the figure, ABC, BED and FEC are straight lines. BD bisects $\angle CBF$. $BC = CD$, $\angle ABF = 2\angle AFB$, $BE = 11$ cm, $\angle ACF = 53°$ and $\angle CAF = 33°$.

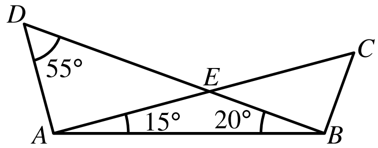


Explain **(a)** Is BF parallel to CD? Explain your answer.

(b) Find the lengths of AF and CD.

Explain **(c)** Suppose BD is a chord of a circle with centre at C. Does the circle pass through F? Explain your answer.

38. In the figure, A, B, C and D are concyclic. AC and BD intersect at E. $\angle ABD = 20°$, $\angle ADB = 55°$ and $\angle BAC = 15°$.

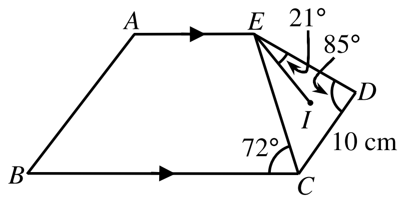


(a) Prove that the line segment joining C and D is a diameter of the circle passing through A, B, C and D.

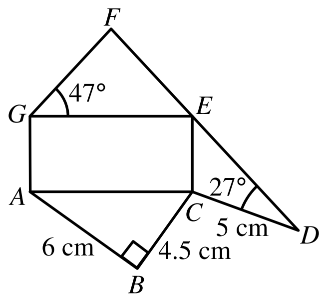
Explain **(b)** Someone claims that $\dfrac{\text{area of } \triangle ADE}{\text{area of } \triangle BCE} = \dfrac{\sin^2 20°}{\sin^2 15°}$. Do you agree?
Explain your answer.

(c) If $AD = 6$ cm, find the perimeter of the polygon $ABCED$.

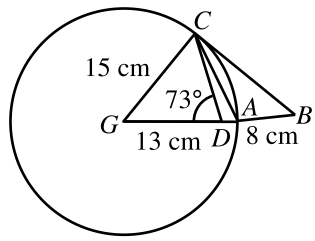
* **39.** In the figure, $ABCDE$ is a pentagon, where $AE = DE$ and $AE \parallel BC$. I is the in-centre of $\triangle CDE$. $\angle ABC = \angle DCE$, $CD = 10$ cm, $\angle BCE = 72°$, $\angle CDE = 85°$ and $\angle DEI = 21°$. Find the perimeter of $ABCDE$.



* **40.** In the figure, DEF is a straight line. The area of the rectangle $ACEG$ is 30 cm^2. $AB = 6$ cm, $BC = 4.5$ cm, $CD = 5$ cm, $\angle ABC = 90°$, $\angle CDE = 27°$ and $\angle EGF = 47°$. Find the area of the polygon $ABCDFG$.



* **41.** In the figure, G is the centre of the circle and the radius of the circle is 15 cm. D is a point on GA such that $DG = 13$ cm and $\angle CDG = 73°$. BC is the tangent to the circle at C. $\angle ABC$ is an acute angle and $AB = 8$ cm.



(a) Find $\angle CGD$.
Hence, find the length of the CD.

(b) Solve $\triangle ABC$.

Answers

Consolidation Exercise 9A

1. 6.13

2. 6.91

3. 3.85

4. 5.90

5. 2.07

6. 4

7. $B = 57°$, $AC = 26.0$ cm, $BC = 17.8$ cm

8. $C = 35°$, $AC = 12.4$ cm, $BC = 27.8$ cm

9. $B = 52°$, $AB = 27.9$ cm, $BC = 18.3$ cm

10. 40.1°

11. 33.5°

12. 71.0°

13. 30°

14. 76.8°

15. 135°

16. no

17. yes, 32.7°

18. no

19. no

20. **(a)** 48° **(b)** 68.3°

21. **(a)** 6° **(b)** 1.91 cm

22. **(a)** 34.8° **(b)** no

23. **(a)** 7.69 cm **(b)** 26.7 cm

24. **(a)** 69.2° *or* 111° **(b)** 85.8° *or* 94.2°
(c) 23.9°

25. **(a)** yes, $B = 136°$, $a = 25.2$ cm, $b = 34.0$ cm
(b) yes, $N = 86°$, $\ell = 3.87$ cm, $m = 2.59$ cm
(c) yes, $D = 50°$, $e = 22.4$ cm, $f = 11.1$ cm

26. **(a)** no
(b) yes, $P = 62.1°$, $R = 18.9°$, $r = 6.23$ cm
(c) yes, $R = 111°$, $S = 48.0°$, $r = 10.7$ cm *or*
$R = 27.0°$, $S = 132°$, $r = 5.19$ cm
(d) yes, $X = 66.7°$, $Z = 62.3°$, $z = 6.27$ cm
or
$X = 113°$, $Z = 15.7°$, $z = 1.92$ cm

27. **(a)** $DF = 10.5$ cm *or* $DF \geq 21$ cm
(b) 10.5 cm $< DF <$ 21 cm

28. **(a)** 19.5° **(b)** 1.52

29. **(a)** $\sqrt{3} : 1 : 1$ **(b)** 19.4 cm

30. **(a)** 9.53 cm **(b)** 43.5 cm

31. **(a)** 69.5° **(b)** 30.6 cm

32. **(a)** 9.61 cm **(b)** 11.6 cm

33. **(a)** 10.2 cm **(b)** $\triangle ADC$

34. **(a)** 33.3° **(b)** 16.4 cm

35. **(a)** 14.4 cm **(b)** 10.1 cm

36. **(a)** $BC = 7.74$ cm, $CD = 5.97$ cm
(b) 47.3 cm^2

37. **(a)** yes
(b) $AF = 28.2$ cm, $CD = 13.7$ cm
(c) no

38. **(b)** yes **(c)** 43.3 cm

39. 78.8 cm

40. 69.6 cm^2

41. **(a)** $\angle CGD = 51.0°$, $CD = 12.2$ cm
(b) $\angle ABC = 44.1°$, $\angle ACB = 25.5°$,
$\angle BAC = 110°$, $AC = 12.9$ cm,
$BC = 17.4$ cm

F5B: Chapter 9B

Date	Task	Progress		
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Book Example 7	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 8	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 9	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 10	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Book Example 11	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)	
	Consolidation Exercise	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)	
	Maths Corner Exercise 9B Level 1	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 9B Level 2	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 9B Multiple Choice	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	Mark: _________	

5B Lesson Worksheet 9.2A

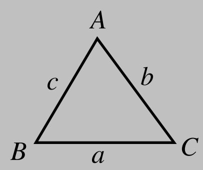
(Refer to Book 5B P.9.18)

Objective: To use the cosine formula to find unknown side/angle when two sides and their included angle are given.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

Cosine Formula

$a^2 = b^2 + c^2 - 2bc\cos A$	$b^2 = c^2 + a^2 - 2ca\cos B$	$c^2 = a^2 + b^2 - 2ab\cos C$
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	$\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



Given Two Sides and Their Included Angle (*SAS*)

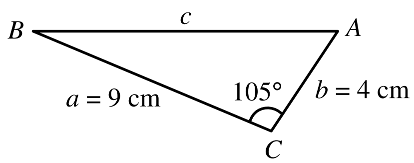
If two sides and their included angle of a triangle are given, the triangle can be solved by the following steps.

(i) Use the cosine formula to find the remaining side.

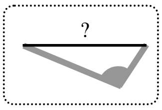
(ii) Use the cosine formula / the sine formula and the angle sum of triangle to find the other two angles.

Instant Example 1

In △ ABC, $a = 9$ cm, $b = 4$ cm and $C = 105°$. Solve the triangle.



By the cosine formula, ◂ **Step (i)**



$c^2 = a^2 + b^2 - 2ab\cos C$

$c = \sqrt{9^2 + 4^2 - 2\times9\times4\times\cos105°}$ cm

$= \underline{10.8 \text{ cm}}$, *cor. to 3 sig. fig.* 10.753

By the cosine formula, ◂ **Step (ii)**

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{4^2 + 10.753^2 - 9^2}{2\times4\times10.753}$

$A = \underline{53.9°}$, *cor. to 3 sig. fig.* 53.948

By the sine formula,

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{9\text{ cm}}{\sin A} = \dfrac{10.753\text{ cm}}{\sin105°}$

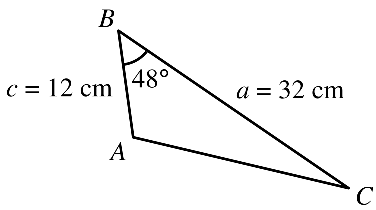
$A + B + C = 180°$ (*∠ sum of △*)

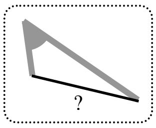
$53.948° + B + 105° = 180°$

$B = \underline{21.1°}$, *cor. to 3 sig. fig.*

Instant Practice 1

In △ ABC, $a = 32$ cm, $c = 12$ cm and $B = 48°$. Solve the triangle.





By the cosine formula,

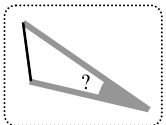
$b^2 = (\quad)^2 + (\quad)^2 - 2(\quad)(\quad)\cos(\quad)$

$b = [(\quad)^2 + (\quad)^2 -$

$2\times(\quad)\times(\quad)\times\cos(\quad)]^{\frac{1}{2}}$ cm

$=$ ________, *cor. to 3 sig. fig.* [ ]

By the cosine formula,

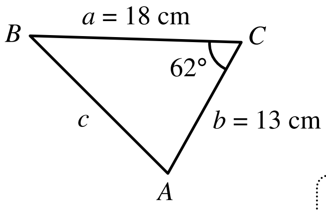


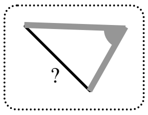
$\cos C = \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}$

$= \dfrac{(\quad)^2 + (\qquad)^2 - (\quad)^2}{2\times(\quad)\times(\qquad)}$

$C =$ ______, *cor. to 3 sig. fig.* [ ]

1. In △ ABC, $a = 18$ cm, $b = 13$ cm and $C = 62°$. Solve the triangle.





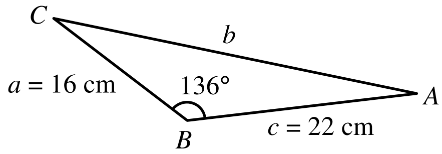
By the cosine formula,

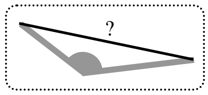
$c^2 = (\quad)^2 + (\quad)^2 - 2(\quad)(\quad)\cos(\quad)$

You may find A or B in the second step.

2. In △ ABC, $a = 16$ cm, $c = 22$ cm and $B = 136°$. Solve the triangle.

➥Ex 9B: 10, 11



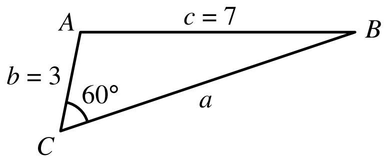


⇪Level Up Question⇪

3. In △ ABC, $b = 3$, $c = 7$, $C = 60°$ and A is an obtuse angle.

(a) Find a by the cosine formula.

(b) Find a by the sine formula.



5B Lesson Worksheet 9.2B

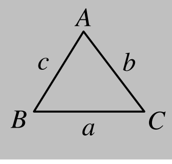
(Refer to Book 5B P.9.20)

Objective: To use the cosine formula to find unknown side/angle when three sides are given.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

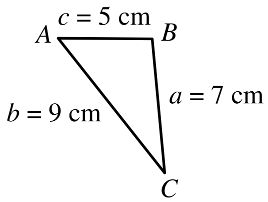
Cosine Formula: Given Three Sides (*SSS*)

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ $\quad\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$ $\quad\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



Instant Example 1

In △ *ABC*, $a = 7$ cm, $b = 9$ cm and $c = 5$ cm. Solve the triangle.



By the cosine formula,

$\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

$= \dfrac{5^2 + 7^2 - 9^2}{2 \times 5 \times 7}$

The largest angle is opposite to the longest side. Find the largest angle first. Then the remaining two angles must be acute.

B = <u>95.7°</u>, *cor. to 3 sig. fig.* [95.739]

By the cosine formula,

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{7^2 + 9^2 - 5^2}{2 \times 7 \times 9}$

C = <u>33.6°</u>, *cor. to 3 sig. fig.* [33.557]

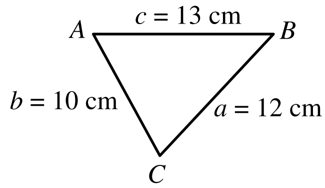
$A + B + C = 180°$ (*∠ sum of △*)

$A + 95.739° + 33.557° = 180°$

A = <u>50.7°</u>, *cor. to 3 sig. fig.*

Instant Practice 1

In △ *ABC*, $a = 12$ cm, $b = 10$ cm and $c = 13$ cm. Solve the triangle.



By the cosine formula,

$\cos C = \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}$

$= \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2 \times (\quad) \times (\quad)}$

C = ______, *cor. to 3 sig. fig.* [ ]

By the cosine formula,

$\cos B = \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}$

$= \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2 \times (\quad) \times (\quad)}$

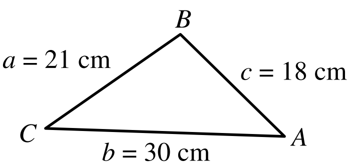
B = ______, *cor. to 3 sig. fig.* [ ]

$A + B + C = 180°$

$A + (\qquad) + (\qquad) = 180°$

A = ______, *cor. to 3 sig. fig.*

1.

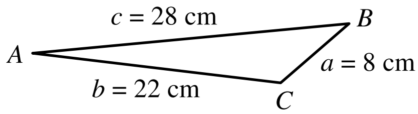


In △ *ABC*, $a = 21$ cm, $b = 30$ cm and $c = 18$ cm. Find *A*.

By the cosine formula,

$\cos A = \dfrac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}$

2.

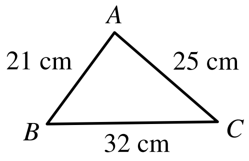


In △ *ABC*, $a = 8$ cm, $b = 22$ cm and $c = 28$ cm. Find *B*.

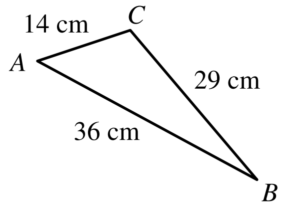
➥Ex 9B: 5–9

Solve the following triangles. **[Nos. 3–4]** ➥Ex 9B: 12

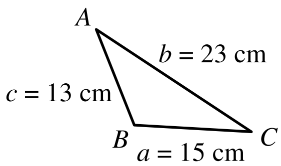
3.



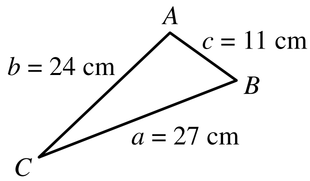
4.



5. In △ *ABC*, a = 15 cm, b = 23 cm and c = 13 cm. Find the largest angle of △ *ABC*.



6. In △ *ABC*, a = 27 cm, b = 24 cm and c = 11 cm. Find the smallest angle of △ *ABC*.



➥Ex 9B: 15, 16

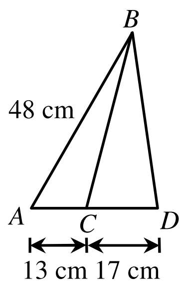
In a triangle, the smallest angle is opposite to the shortest side.

⇧Level Up Question⇧

7. In the figure, *ACD* is a straight line. *AB* = 48 cm, *AC* = 13 cm, *BC* = 43 cm and *CD* = 17 cm.

(a) Find ∠*BAC*.

(b) Hence, find *BD*.



9 Solving Triangles

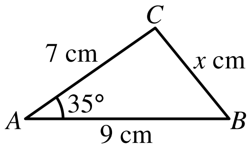
✂ Consolidation Exercise 9B

[*In this exercise, give the answers correct to 3 significant figures if necessary.*]

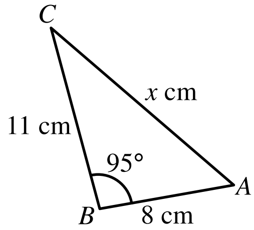
Level 1

In each of the following triangles, find the unknown. **[Nos. 1–9]**

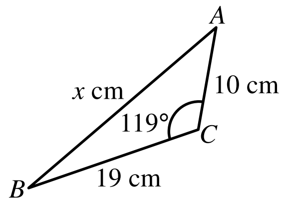
1.



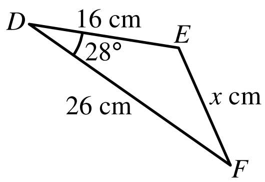
2.



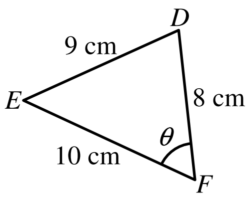
3.



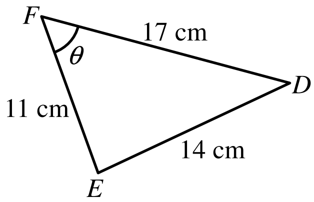
4.



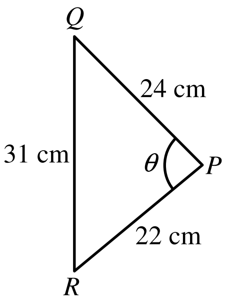
5.



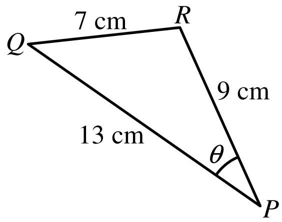
6.



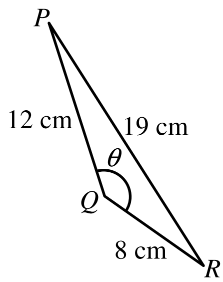
7.



8.

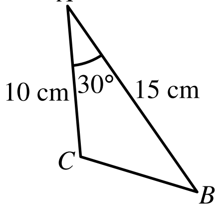


9.

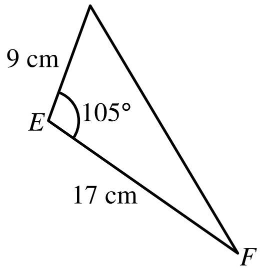


Solve the following triangles. **[Nos. 10–12]**

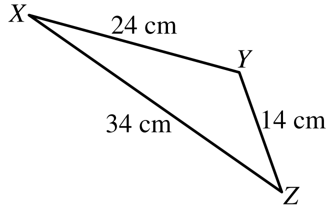
10.



11.



12.



13. In each of the following, find a in $\triangle ABC$.

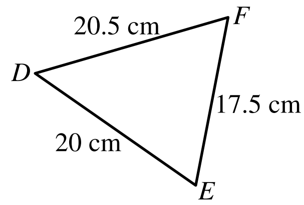
(a) $b = 9$ cm, $c = 11$ cm, $A = 45°$ **(b)** $b = 6$ cm, $c = 14$ cm, $A = 70°$

(c) $b = 37$ cm, $c = 23$ cm, $A = 106°$ **(d)** $b = 15$ cm, $c = 28$ cm, $A = 123°$

14. In each of the following, find Q in $\triangle PQR$.

(a) $p = 10$ cm, $q = 15$ cm, $r = 13$ cm **(b)** $p = 22$ cm, $q = 26$ cm, $r = 11$ cm

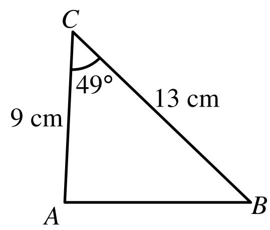
15. In the figure, $EF = 17.5$ cm, $DF = 20.5$ cm and $DE = 20$ cm. Find the largest angle of $\triangle DEF$.



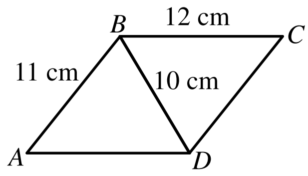
16. In the figure, $AC = 9$ cm, $BC = 13$ cm and $\angle ACB = 49°$.

(a) Find the length of AB.

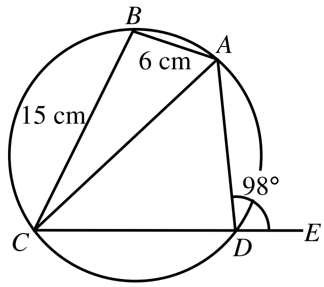
(b) Find the smallest angle of $\triangle ABC$.



17. In the figure, $ABCD$ is a parallelogram. $AB = 11$ cm, $BC = 12$ cm and $BD = 10$ cm. Find $\angle ADC$.



18. In the figure, A, B, C and D are points on a circle. E is a point on CD produced. $\angle ADE = 98°$, $AB = 6$ cm and $BC = 15$ cm. Find the length of AC.



Level 2

19. In each of the following, find the smallest angle of the triangle.

(a) In $\triangle ABC$, $a = 4$ cm, $b = 11$ cm and $c = 9$ cm.

(b) In $\triangle XYZ$, $x = 18$ cm, $y = 21$ cm and $z = 24$ cm.

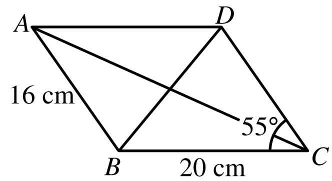
20. In each of the following, solve the triangle.

(a) In $\triangle ABC$, $a = 32$ cm, $c = 23$ cm and $B = 131°$.

(b) In $\triangle DEF$, $e = 14$ cm, $f = 8$ cm and $D = 53°$.

(c) In $\triangle PQR$, $p = 32$ cm, $q = 46$ cm and $r = 29$ cm.

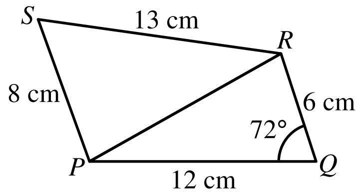
21. In the figure, $ABCD$ is a parallelogram. $AB = 16$ cm, $BC = 20$ cm and $\angle BCD = 55°$. Find the lengths of the two diagonals BD and AC.



22. In the figure, $PQ = 12$ cm, $PS = 8$ cm, $QR = 6$ cm, $RS = 13$ cm and $\angle PQR = 72°$.

(a) Find the length of PR.

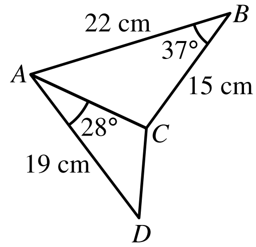
(b) Find $\angle PRS$.



23. In the figure, $AB = 22$ cm, $AD = 19$ cm, $BC = 15$ cm, $\angle ABC = 37°$ and $\angle CAD = 28°$.

(a) Find the lengths of AC and CD.

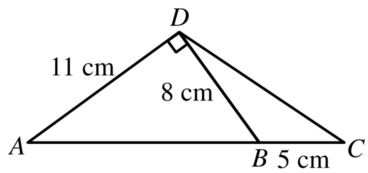
(b) Find $\angle ACD$.



24. In the figure, ABC is a straight line. $AD = 11$ cm, $BC = 5$ cm, $BD = 8$ cm and $\angle ADB = 90°$.

(a) Find the length of CD.

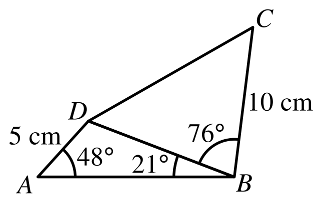
(b) Find $\angle BDC$.



25. In the figure, $AD = 5$ cm, $BC = 10$ cm, $\angle ABD = 21°$, $\angle BAD = 48°$ and $\angle CBD = 76°$.

(a) Find the length of BD.

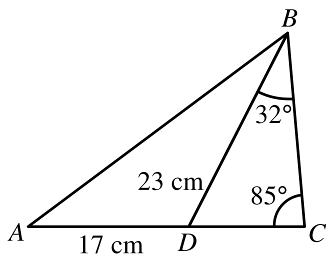
(b) Find the perimeter of the quadrilateral $ABCD$.



26. In the figure, ADC is a straight line. $AD = 17$ cm, $BD = 23$ cm, $\angle BCD = 85°$ and $\angle CBD = 32°$.

(a) Find the length of AB.

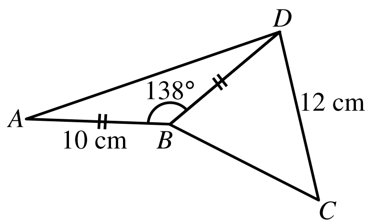
(b) Find the perimeter of $\triangle ABC$.



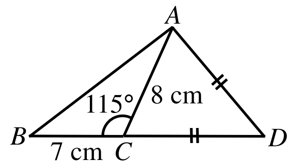
27. In the figure, $\angle ADC = 4\angle ADB$, $AB = BD = 10$ cm, $CD = 12$ cm and $\angle ABD = 138°$.

(a) Find $\angle BDC$.

Explain **(b)** Is the perimeter of the polygon $ABCD$ greater than 55 cm? Explain your answer.



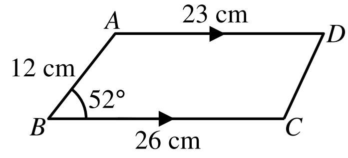
28. In the figure, $BC = 7$ cm, $AC = 8$ cm and $\angle ACB = 115°$. BC is produced to the point D such that $CD = AD$.



(a) Find the length of AB.

Explain **(b)** Determine whether $\triangle ABD$ is an acute-angled triangle, a right-angled triangle or an obtuse-angled triangle. Explain your answer.

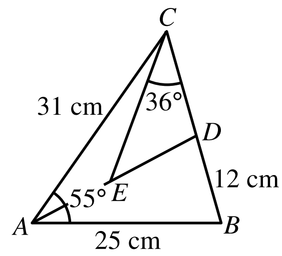
29. In the figure, $ABCD$ is a trapezium, where AD // BC. $AB = 12$ cm, $AD = 23$ cm, $BC = 26$ cm and $\angle ABC = 52°$.



(a) Find $\angle ADC$.

(b) Find the perimeter of $ABCD$.

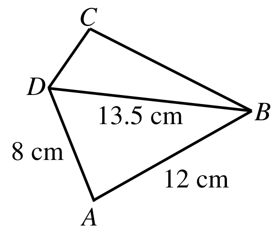
30. In the figure, $AB = 25$ cm, $AC = 31$ cm and $\angle BAC = 55°$. D is a point on BC such that $BD = 12$ cm. E is a point on AD such that $\angle DCE = 36°$.



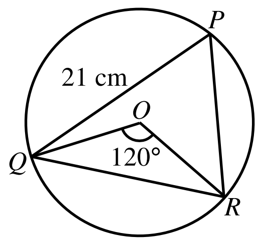
(a) Find the lengths of CD and AD.

(b) Find $\angle AEC$.

31. In the figure, A, B, C and D are concyclic. BD is the angle bisector of $\angle ADC$. $AB = 12$ cm, $AD = 8$ cm and $BD = 13.5$ cm. Find the perimeter of the quadrilateral $ABCD$.



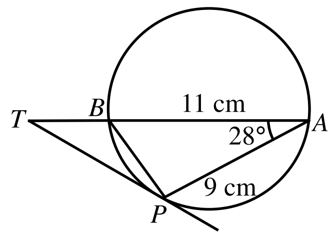
32. In the figure, P, Q and R are points on the circle with centre O. $PQ = 21$ cm, $QR = (x + 2)$ cm, $PR = (x - 1)$ cm and $\angle QOR = 120°$.



(a) Find the value of x.

(b) Find $\angle OQP$.

33. In the figure, A, B and P are points on the circle. TP touches the circle at P. ABT is a straight line. $AB = 11$ cm, $AP = 9$ cm and $\angle BAP = 28°$.

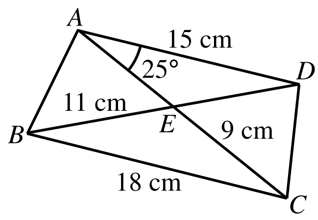


(a) Find the length of BP.

(b) Find the length of BT.

(c) Find the radius of the circle.

34. In the figure, AC and BD intersect at the point E. $AD = 15$ cm, $BC = 18$ cm, $BE = 11$ cm, $CE = 9$ cm and $\angle CAD = 25°$.

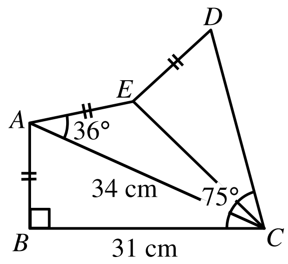


(a) Find the lengths of DE and AE.

Explain **(b)** Is AD parallel to BC? Explain your answer.

(c) Find the perimeter of the quadrilateral $ABCD$.

35. In the figure, $\triangle ABC$ is a right-angled triangle and $\triangle CDE$ is an acute-angled triangle. $AB = AE = DE$, $AC = 34$ cm, $BC = 31$ cm, $\angle BCD = 75°$ and $\angle CAE = 36°$.



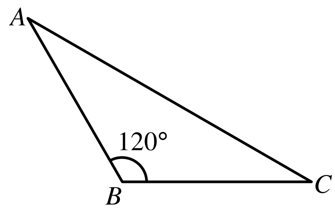
(a) Find the length of AB.

(b) Find $\angle ACE$.

(c) Find reflex $\angle AED$.

36. **(a)** Let $f(x) = x^2 - 15x + 225$. Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$.

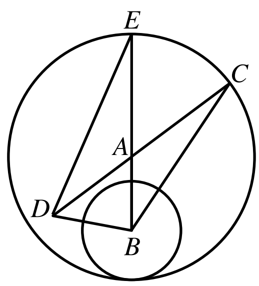
(b) In the figure, $AB + BC = 15$ cm and $\angle ABC = 120°$. Let $AB = p$ cm.



(i) Express the length of AC in terms of p.

Explain **(ii)** Can the perimeter of $\triangle ABC$ be less than 27 cm? Explain your answer.

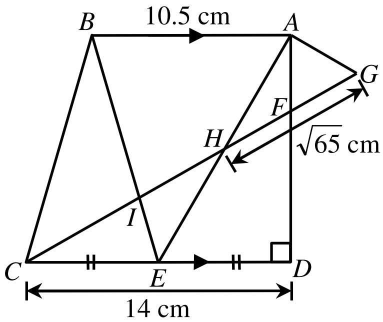
* **37.** In the figure, A and B are the centres of circles with radii 5 cm and 2 cm respectively. The two circles touch each other internally. C and E are points on the larger circle. BE and CD intersect at A. $AD = 4$ cm and $BC = \sqrt{52}$ cm.



(a) Find the lengths of BD and DE.

Explain **(b)** Are B, C, D and E concyclic? Explain your answer.

* **38.** In the figure, $ABCD$ is a trapezium, where BA // CD, $BA = 10.5$ cm, $CD = 14$ cm and $\angle ADC = 90°$. The area of $ABCD$ is 147 cm^2. E is the mid-point of CD and $BC = BE$. CG intersects AD, AE and BE at F, H and I respectively, where $AF : FD = 1 : 2$ and $GH = \sqrt{65}$ cm.



(a) Find the lengths of AE and BC.

(b) Find $\angle AEB$ and $\angle GIE$.

(c) Solve $\triangle AGH$.

Answers

Consolidation Exercise 9B

1. 5.18 **2.** 14.2

3. 25.4 **4.** 14.0

5. 58.8° **6.** 55.1°

7. 84.6° **8.** 30.8°

9. 143°

10. $B = 38.3°$, $C = 112°$, $BC = 8.07$ cm

11. $D = 50.8°$, $F = 24.2°$, $DF = 21.2$ cm

12. $X = 19.7°$, $Y = 125°$, $Z = 35.4°$

13. **(a)** 7.87 cm **(b)** 13.2 cm
(c) 48.7 cm **(d)** 38.3 cm

14. **(a)** 80.3° **(b)** 98.4°

15. 65.9°

16. **(a)** 9.82 cm **(b)** 43.7°

17. 129°

18. 16.9 cm

19. **(a)** 20.0° **(b)** 46.6°

20. **(a)** $A = 28.8°$, $C = 20.2°$, $b = 50.2$ cm
(b) $E = 92.2°$, $F = 34.8°$, $d = 11.2$ cm
(c) $P = 43.6°$, $Q = 97.8°$, $R = 38.7°$

21. $BD = 17.0$ cm, $AC = 32.0$ cm

22. **(a)** 11.6 cm **(b)** 37.4°

23. **(a)** $AC = 13.5$ cm, $CD = 9.51$ cm
(b) 110°

24. **(a)** 11.7 cm **(b)** 20.3°

25. **(a)** 10.4 cm **(b)** 40.6 cm

26. **(a)** 34.2 cm **(b)** 84.1 cm

27. **(a)** 63° **(b)** no

28. **(a)** 12.7 cm
(b) obtuse-angled triangle

29. **(a)** 65.1° **(b)** 71.4 cm

30. **(a)** $CD = 14.4$ cm, $AD = 24.6$ cm
(b) 138°

31. 36.8 cm

32. **(a)** 17 **(b)** 16.8°

33. **(a)** 5.21 cm **(b)** 5.55 cm
(c) 5.55 cm

34. **(a)** $DE = 8.05$ cm, $AE = 8.64$ cm
(b) no **(c)** 49.4 cm

35. **(a)** 14.0 cm **(b)** 19.9°
(c) 211°

36. **(a)** $\left(\frac{15}{2}, \frac{675}{4}\right)$
(b) **(i)** $\sqrt{p^2 - 15p + 225}$ cm
(ii) no

37. **(a)** $BD = 3.26$ cm, $DE = 8.06$ cm
(b) no

38. **(a)** $AE = 13.9$ cm, $BC = 12.5$ cm
(b) $\angle AEB = 46.5°$, $\angle GIE = 103°$
(c) $\angle AGH = 59.5°$, $\angle AHG = 30.0°$, $\angle GAH = 90.5°$, $AG = 4.03$ cm, $AH = 6.95$ cm

F5B: Chapter 9C			
Date	Task	Progress	
	Lesson Worksheet	◯ Complete and Checked ◯ Problems encountered ◯ Skipped	(Full Solution)
	Book Example 12	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 13	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 14	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 15	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 16	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 17	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)
	Book Example 18	◯ Complete ◯ Problems encountered ◯ Skipped	(Video Teaching)

	Consolidation Exercise	○ Complete and Checked ○ Problems encountered ○ Skipped	(Full Solution)	
	Maths Corner Exercise 9C Level 1	○ Complete and Checked ○ Problems encountered ○ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 9C Level 2	○ Complete and Checked ○ Problems encountered ○ Skipped	Teacher's Signature	___________ ( )
	Maths Corner Exercise 9C Multiple Choice	○ Complete and Checked ○ Problems encountered ○ Skipped	Teacher's Signature	___________ ( )
	E-Class Multiple Choice Self-Test	○ Complete and Checked ○ Problems encountered ○ Skipped	Mark: _________	

✂ 5B Lesson Worksheet 9.3A

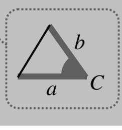
(Refer to Book 5B P.9.26)

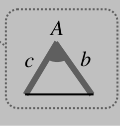
Objective: To use the formula $\frac{1}{2}ab\sin C$ to find areas of triangles.

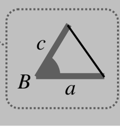
[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

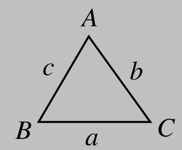
Finding the Area of a Triangle Given Two Sides and Their Included Angle

Area of $\triangle ABC = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$



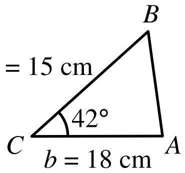






Instant Example 1

In the figure, find the area of $\triangle ABC$.



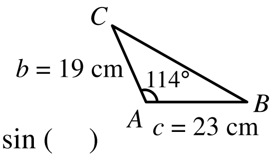
Area of $\triangle ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 15 \times 18 \times \sin 42°\ \text{cm}^2$

$=$ 90.3 cm², *cor. to 3 sig. fig.*

Instant Practice 1

In the figure, find the area of $\triangle ABC$.

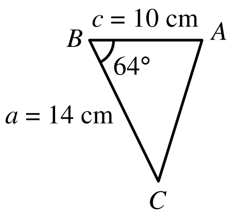


Area of $\triangle ABC = \frac{1}{2}(\quad)(\quad)\sin(\quad)$

$= \frac{1}{2} \times (\quad) \times (\quad) \times \sin(\quad)\ \text{cm}^2$

$=$ __________, *cor. to 3 sig. fig.*

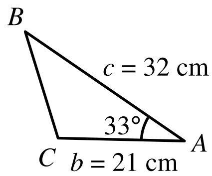
1. In the figure, find the area of $\triangle ABC$.



Area of $\triangle ABC$

$= \frac{1}{2}(\quad)(\quad)\sin(\quad)$

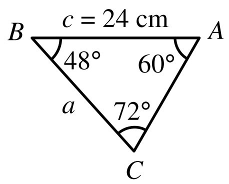
2. In the figure, find the area of $\triangle ABC$.



➥Ex 9C: 1

Instant Example 2

In the figure, find a and the area of $\triangle ABC$.

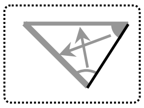


By the sine formula,

$\frac{a}{\sin 60°} = \frac{24\text{ cm}}{\sin 72°}$

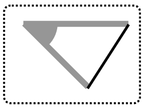
$a = \frac{24\sin 60°}{\sin 72°}$ cm

$=$ 21.9 cm, *cor. to 3 sig. fig.*



21.854

Area of $\triangle ABC = \frac{1}{2}ca\sin B$

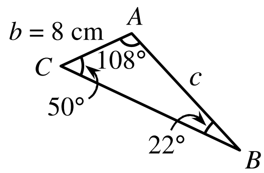


$= \frac{1}{2} \times 24 \times 21.854 \times \sin 48°\ \text{cm}^2$

$=$ 195 cm², *cor. to 3 sig. fig.*

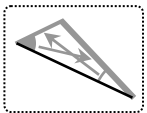
Instant Practice 2

In the figure, find c and the area of $\triangle ABC$.



By the sine formula,

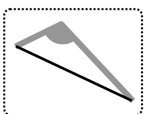
$\frac{c}{\sin(\quad)} = \frac{(\quad)}{\sin(\quad)}$



$c = \frac{(\quad)(\quad)}{(\quad)}$ cm

$=$ __________, *cor. to 3 sig. fig.*

Area of $\triangle ABC = \frac{1}{2}(\quad)(\quad)\sin(\quad)$



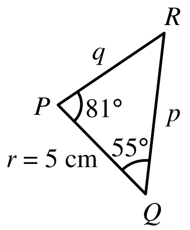
$=$

In each of the following, find the area of the triangle. **[Nos. 3–6]** ➥Ex 9C: 7–9

3. $P + Q + R = 180°$

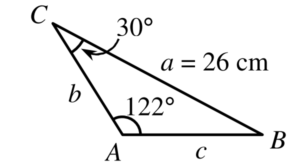
$(\quad) + (\quad) + R = 180°$

$R = (\quad)$



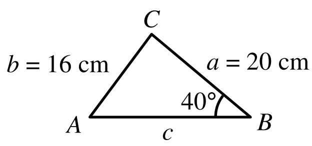
Find p or q.

4.



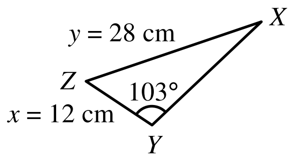
Need to find only one of b and c.

5. A is acute.



Steps:
1. Use the sine formula to find A.
2. Find C.
3. Use $\frac{1}{2}ab \sin C$ to find the area.

6.



⇪Level Up Question⇪

7. In $\triangle ABC$, $AB = 5$ cm, $AC = 8$ cm and $\angle BAC = \theta$, where $0° < \theta < 180°$.

(a) Find the area of $\triangle ABC$ when

(i) $\theta = 30°$, **(ii)** $\theta = 120°$.

Explain **(b)** Someone claims that the maximum area of $\triangle ABC$ is 20 cm^2. Do you agree? Explain your answer.

5B Lesson Worksheet 9.3B

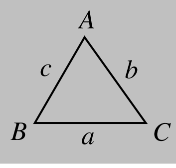
(Refer to Book 5B P.9.30)

Objective: To use Heron's formula to find areas of triangles.

[*In this worksheet, give the answers correct to 3 significant figures if necessary.*]

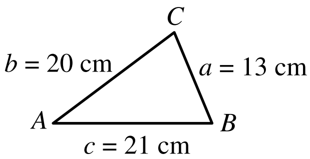
Find the Area of a Triangle Given Three Sides — Heron's Formula

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$.



Instant Example 1

In the figure, a = 13 cm, b = 20 cm and c = 21 cm. Find the area of $\triangle ABC$.



$s = \frac{1}{2}(13 + 20 + 21)$ cm

$= 27$ cm

By Heron's formula,

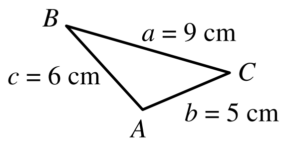
area of $\triangle ABC$

$= \sqrt{27(27-13)(27-20)(27-21)}$ cm^2

$= \underline{\underline{126 \text{ cm}^2}}$

Instant Practice 1

In the figure, a = 9 cm, b = 5 cm and c = 6 cm. Find the area of $\triangle ABC$.



$s = \frac{1}{2}[(\quad) + (\quad) + (\quad)]$ cm

$= (\quad)$ cm

By Heron's formula,

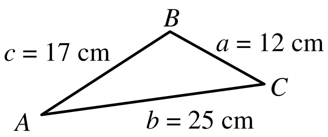
area of $\triangle ABC$

$= \sqrt{(\quad)(\quad)(\quad)(\quad)}$ cm^2

= ____________, *cor. to 3 sig. fig.*

In each of the following, find the area of the triangle. **[Nos. 1–4]** ➥Ex 9C: 11

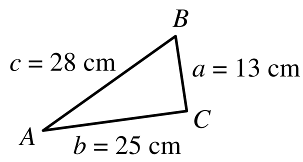
1.



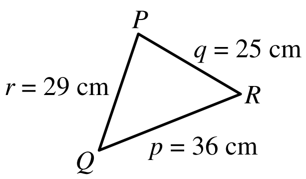
$s = \frac{1}{2}[(\quad) + (\quad) + (\quad)]$ cm

$= (\quad)$ cm

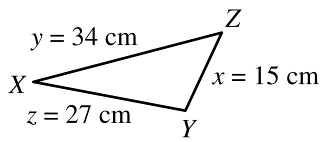
2.



3.

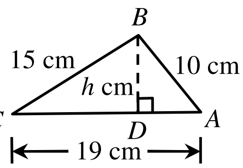


4.



Instant Example 2

In the figure, find the area of $\triangle ABC$ and the value of h.



$s = \frac{1}{2}(15+19+10)$ cm = 22 cm

By Heron's formula,

area of $\triangle ABC$

$= \sqrt{22(22-15)(22-19)(22-10)}$ cm^2

$= \underline{\underline{74.5 \text{ cm}^2}}$, *cor. to 3 sig. fig.* $\boxed{74.458}$

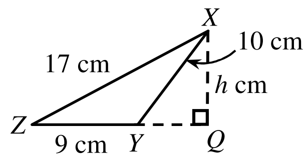
Area of $\triangle ABC = \frac{1}{2} \times AC \times BD$

$74.458 = \frac{1}{2} \times 19 \times h$

$h = \underline{\underline{7.84}}$, *cor. to 3 sig. fig.*

Instant Practice 2

In the figure, find the area of $\triangle XYZ$ and the value of h.



$s = \frac{1}{2}$[(   ) + (   ) + (   )] cm = (   ) cm

By Heron's formula,

area of $\triangle XYZ$

$= \sqrt{(\quad)(\quad)(\quad)(\quad)}$ cm^2

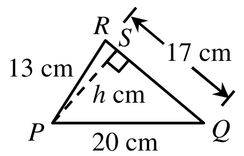
= ________

Area of $\triangle XYZ = \frac{1}{2} \times$ (   ) $\times XQ$

(   ) $= \frac{1}{2} \times$ (   ) $\times h$

h = ______

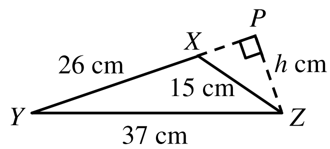
5. In the figure, find the area of $\triangle PQR$ and the value of h.



$s = \frac{1}{2}$[(   ) + (   ) + (   )] cm = (   ) cm

By Heron's formula,

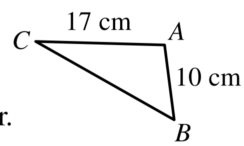
6. In the figure, find the area of $\triangle XYZ$ and the value of h.



➥Ex 9C: 12, 13

⇧Level Up Question⇧

7. In the figure, the perimeter of $\triangle ABC$ is 48 cm.



(a) Using Heron's formula, find the area of $\triangle ABC$.

Explain **(b)** Is the shortest distance between A and BC greater than 8 cm? Explain your answer.

9 Solving Triangles

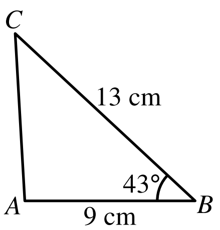
✂ Consolidation Exercise 9C

[*In this exercise, give the answers correct to 3 significant figures if necessary.*]

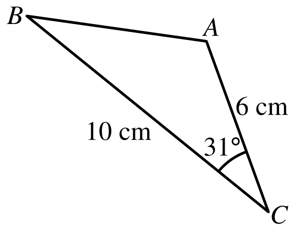
Level 1

1. In each of the following, find the area of the triangle.

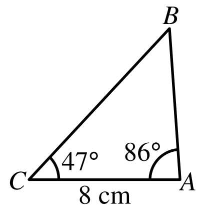
(a)



(b)

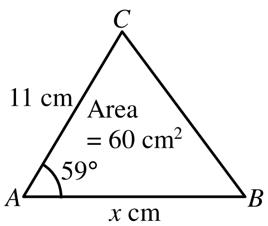


(c)

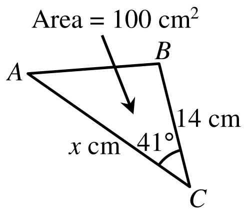


In each of the following triangles, find the unknown. **[Nos. 2–3]**

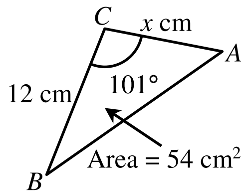
2. **(a)**



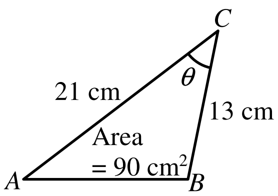
(b)



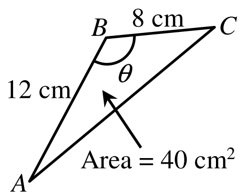
(c)



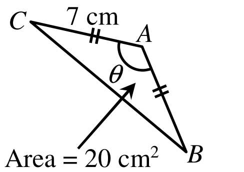
3. **(a)** θ is an acute angle.



(b) θ is an obtuse angle.

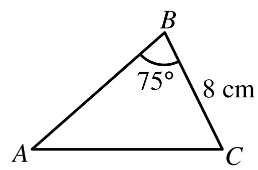


(c) θ is an obtuse angle.



4. The area of $\triangle ABC$ is 24 cm^2 and C is an acute angle. If a = 12 cm and b = 6 cm, find C.

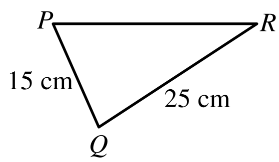
5. In the figure, $\angle ABC = 75°$ and BC = 8 cm. The area of $\triangle ABC$ is 42 cm^2.



(a) Find the length of AB.

(b) Find the length of AC.

6. In the figure, $\triangle PQR$ is an acute-angled triangle. PQ = 15 cm and QR = 25 cm. The area of $\triangle PQR$ is 185 cm^2.



(a) Find $\angle PQR$.

(b) Find the length of PR.

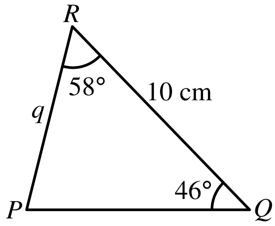
7. The lengths of the three sides of a triangle are 19 cm, 15 cm and x cm, where $4 < x < 15$. The area of the triangle is 91 cm^2.

(a) Find the smallest angle of the triangle.

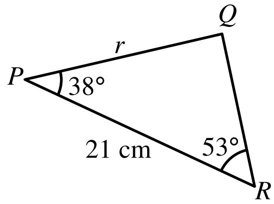
(b) Find the value of x.

In each of the following triangles, find the unknown and the area of $\triangle PQR$. **[Nos. 8–10]**

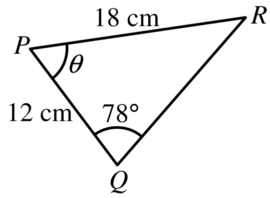
8.



9.

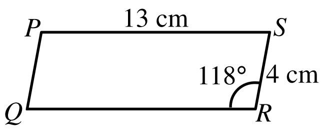


10.

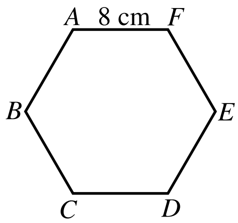


11. In each of the following, divide the polygon into triangles and hence find the area of the polygon.

(a) $PQRS$ is a parallelogram.

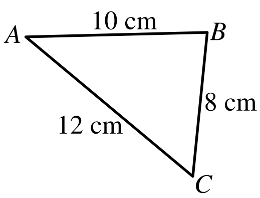


(b) $ABCDEF$ is a regular hexagon.

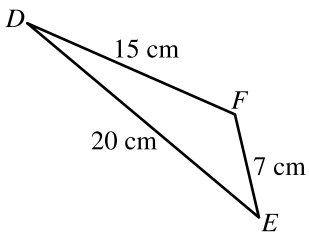


12. In each of the following, find the area of the triangle using Heron's formula.

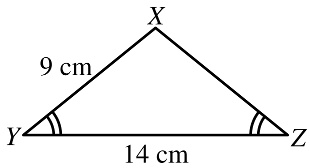
(a)



(b)



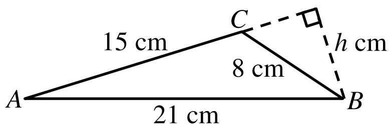
(c)



13. In the figure, $AB = 21$ cm, $AC = 15$ cm and $BC = 8$ cm.

(a) Using Heron's formula, find the area of $\triangle ABC$.

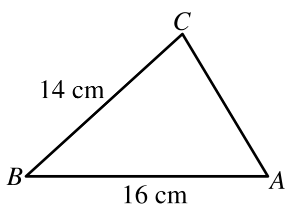
(b) Hence, find the value of h.



14. In the figure, $AB = 16$ cm and $BC = 14$ cm. The perimeter of $\triangle ABC$ is 41 cm.

(a) Using Heron's formula, find the area of $\triangle ABC$.

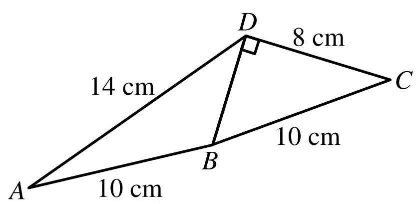
(b) Hence, find the height from B to AC.



15. In the figure, $\angle BDC = 90°$, $AB = BC = 10$ cm, $CD = 8$ cm and $AD = 14$ cm.

(a) Find the area of the quadrilateral $ABCD$.

(b) Find the height of $\triangle ABD$ with AB as the base.



Level 2

16. The area of $\triangle ABC$ is 50 cm^2. If $a = 6$ cm and $c = 19$ cm, find the two possible measures of B.

17. In each of the following, find the area of $\triangle ABC$.

(a) $A = 42°$, $C = 104°$, $a = 5$ m

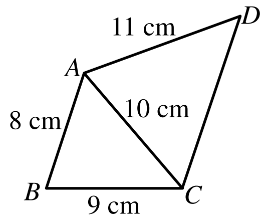
(b) $B = 55°$, $b = 33$ cm, $c = 22$ cm

18. In each of the following, find c.

(a) Area of $\triangle ABC = 25$ m^2, $A = 28°$, $B = 78°$

(b) Area of $\triangle ABC = 32$ cm^2, $A = 32°$, $B = 64°$

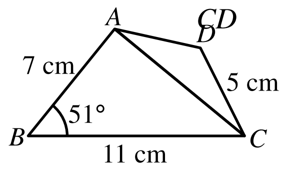
19. The figure shows a quadrilateral $ABCD$ with perimeter of 40 cm. $AB = 8$ cm, $AC = 10$ cm, $AD = 11$ cm and $BC = 9$ cm. Find the area of $ABCD$.



20. In the figure, A, B, C and D are concyclic. $AB = 7$ cm, $BC = 11$ cm, $= 5$ cm and $\angle ABC = 51°$.

(a) Find the length of AC.

(b) Find the area of the quadrilateral $ABCD$.



21. The perimeter of $\triangle PQR$ is 85 cm and $QR : PR : PQ = 3 : 8 : 6$.

(a) Find the lengths of QR, PR and PQ.

(b) Using Heron's formula, find the area of $\triangle PQR$.

(c) Hence, find the height from Q to PR.

22. The area of $\triangle XYZ$ is 56 cm^2 and $X : Y : Z = 21 : 7 : 8$.

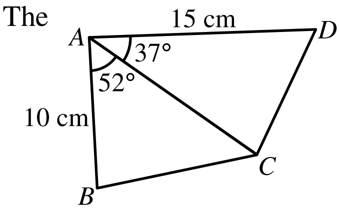
(a) Solve $\triangle XYZ$.

(b) Find the height from X to YZ.

23. In the figure, $AB = 10$ cm, $AD = 15$ cm, $\angle BAC = 52°$ and $\angle CAD = 37°$. The area of the quadrilateral $ABCD$ is 115 cm^2.

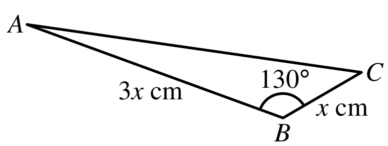
(a) Find the length of AC.

(b) Find $\angle BCD$.

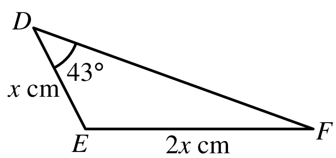


In each of the following, solve the triangle. **[Nos. 24–25]**

24. Area of $\triangle ABC = 84$ cm^2



25. Area of $\triangle DEF = 35$ cm^2



26. In $\triangle ABC$, $AB = 11$ cm, $BC = 21$ cm and $\angle ABC = \theta$.

(a) If the area of $\triangle ABC$ is 75 cm^2, find the possible measures of θ.

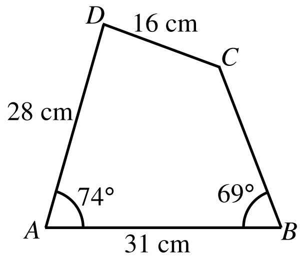
Explain **(b)** **(i)** If the area of $\triangle ABC$ is maximum, what is the measure of θ? Explain your answer.

(ii) Hence, find the maximum area of $\triangle ABC$.

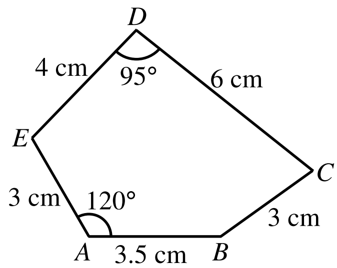
27. In the figure, $\angle BCD$ is an obtuse angle. $AB = 31$ cm, $AD = 28$ cm, $CD = 16$ cm, $\angle ABC = 69°$ and $\angle BAD = 74°$.

(a) Find $\angle BCD$.

(b) Find the area of the quadrilateral $ABCD$.



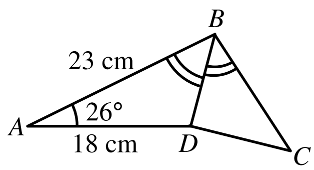
28. In the figure, $AB = 3.5$ cm, $BC = AE = 3$ cm, $CD = 6$ cm, $DE = 4$ cm, $\angle BAE = 120°$ and $\angle CDE = 95°$. Find the area of the pentagon $ABCDE$.



29. In the figure, BD is the angle bisector of $\angle ABC$. $AB = 23$ cm, $AD = 18$ cm and $\angle BAD = 26°$. The ratio of the area of $\triangle ABD$ to that of $\triangle BCD$ is 3 : 2.

(a) Find the area of $\triangle BCD$.

(b) Solve $\triangle BCD$.

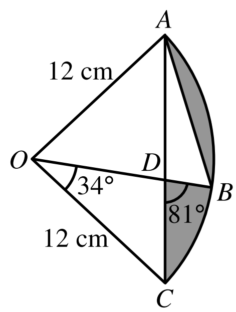


30. In the figure, $OABC$ is a sector with centre O. AC and OB intersect at D. $OA = OB = OC = 12$ cm, $\angle BOC = 34°$ and $\angle BDC = 81°$.

(a) Find the area of $\triangle OAC$.

(b) Find the lengths of AB and AD.

(c) Hence, find the total area of the shaded regions.

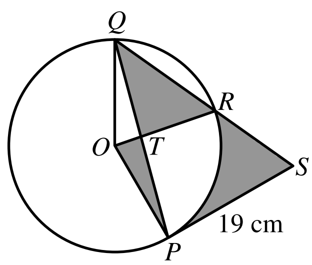


31. In the figure, P, Q and R are points on a circle with centre O. The radius of the circle is 14 cm. PS is the tangent to the circle at P. OTR, PTQ and QRS are straight lines. The area of $\triangle OPQ$ is 49 cm^2. $\angle POQ$ is an obtuse angle and $PS = 19$ cm.

(a) Find the length of QS.

(b) Solve $\triangle QRT$.

(c) Find the total area of the shaded regions.



32. In $\triangle PQR$, $PQ = 8$ cm and $QR = 14$ cm.

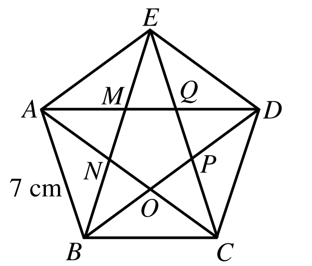
Explain **(a)** Describe how the area of $\triangle PQR$ varies when $\angle PQR$ increases from 40° to 110°. Explain your answer.

(b) Suppose $\angle PQR$ is an obtuse angle and the area of $\triangle PQR$ is 36 cm^2. Find $\angle PQR$.

***33.** In the figure, *ABCDE* and *MNOPQ* are regular pentagons, where $AB = 7$ cm. *AMQD*, *BNME*, *CONA*, *DPOB* and *EQPC* are straight lines.

(a) Find the lengths of *AD* and *PQ*.

(b) Find the areas of *ABCDE* and *MNOPQ*.

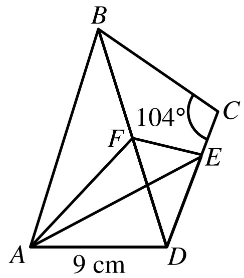


***34.** In the figure, $\triangle BCD$ and $\triangle ABD$ are isosceles triangles, where $BC = CD$ and $AB = BD$. The area of $\triangle ABD$ is 65 cm^2. *F* is the mid-point of *BD*. *E* is a point on *CD* such that $CE : DE = 1 : 2$. $AD = 9$ cm and $\angle BCD = 104°$.

(a) Find the length of *AB* and $\angle BAD$.

(b) Find the area of $\triangle BCD$.

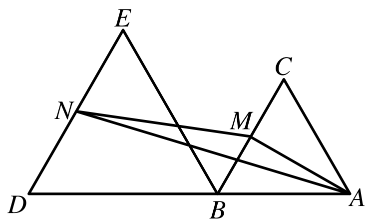
(c) Using Heron's formula, find the area of $\triangle AEF$.



***35.** In the figure, $\triangle ABC$ and $\triangle BDE$ are equilateral triangles, where $AB < BD$. *M* and *N* are the mid-points of *BC* and *DE* respectively. *ABD* is a straight line.

Explain **(a)** Someone claims that the area of $\triangle AMN$ increases when the length of *BD* increases. Do you agree? Explain your answer.

(b) If the area of $\triangle ABC$ is 16 cm^2 and $MN = 9$ cm, find $\angle BMN$.

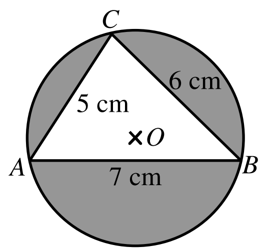


***36.** In the figure, *O* is the circumcentre of $\triangle ABC$. $AB = 7$ cm, $BC = 6$ cm and $AC = 5$ cm.

(a) Find the radius of the circumcircle.

(b) Find the total area of the shaded regions.

(c) Find the perpendicular distance from *O* to *AC*.



***37.** In the figure, *G* is the in-centre of $\triangle ABC$. The radius of the inscribed circle of $\triangle ABC$ is *r* cm. $AB = c$ cm, $AC = b$ cm and $BC = a$ cm.

(a) Express the area of $\triangle BCG$ in terms of *a* and *r*.

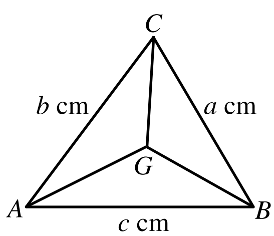
(b) Prove that $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$, where $s = \dfrac{a+b+c}{2}$.

(c) Suppose $a = 16$, $b = 30$ and $c = 32$.

(i) Find the area of $\triangle ABC$.

(ii) Find the radius of the inscribed circle of $\triangle ABC$.

(iii) Find the length of *AG*.



Answers

Consolidation Exercise 9C

1. **(a)** 39.9 cm^2 **(b)** 15.5 cm^2 **(c)** 31.9 cm^2
2. **(a)** 12.7 **(b)** 21.8 **(c)** 9.17
3. **(a)** 41.2° **(b)** 124° **(c)** 125°
4. 41.8°
5. **(a)** 10.9 cm **(b)** 11.7 cm
6. **(a)** 80.6° **(b)** 27.0 cm
7. **(a)** 39.7° **(b)** 12.1
8. q = 7.41 cm, area = 31.4 cm^2
9. r = 16.8 cm, area = 108 cm^2
10. θ = 61.3°, area = 94.7 cm^2
11. **(a)** 45.9 cm^2 **(b)** 166 cm^2
12. **(a)** 39.7 cm^2 **(b)** 42 cm^2 **(c)** 39.6 cm^2
13. **(a)** 46.4 cm^2 **(b)** 6.19
14. **(a)** 75.5 cm^2 **(b)** 13.7 cm
15. **(a)** 50.0 cm^2 **(b)** 5.20 cm
16. 61.3°, 119°
17. **(a)** 10.1 m^2 **(b)** 363 cm^2
18. **(a)** 10.2 m **(b)** 11.6 cm
19. 85.7 cm^2
20. **(a)** 8.55 cm **(b)** 38.6 cm^2
21. **(a)** QR = 15 cm, PR = 40 cm, PQ = 30 cm **(b)** 191 cm^2 **(c)** 9.56 cm
22. **(a)** X = 105°, Y = 35°, Z = 40°, YZ = 17.1 cm, XZ = 10.2 cm, XY = 11.4 cm **(b)** 6.54 cm
23. **(a)** 13.6 cm **(b)** 126°
24. A = 11.9°, C = 38.1°, AB = 25.7 cm, BC = 8.55 cm, AC = 31.8 cm
25. E = 117°, F = 19.9°, DE = 6.27 cm, DF = 16.4 cm, EF = 12.5 cm
26. **(a)** 40.5°, 140° **(b) (i)** 90° **(ii)** 115.5 cm^2
27. **(a)** 131° **(b)** 556 cm^2
28. 24.0 cm^2
29. **(a)** 60.5 cm^2 **(b)** $\angle BCD$ = 42.8°, $\angle BDC$ = 88.0°, $\angle CBD$ = 49.2°, BC = 15.3 cm, BD = 10.4 cm, CD = 11.6 cm
30. **(a)** 71.8 cm^2 **(b)** AB = 10.5 cm, AD = 9.57 cm **(c)** 21.5 cm^2
31. **(a)** 28.7 cm **(b)** $\angle QRT$ = 54.7°, $\angle QTR$ = 85.7°, $\angle RQT$ = 39.7°, QR = 16.2 cm, QT = 13.2 cm, RT = 10.4 cm **(c)** 162 cm^2
32. **(b)** 140°
33. **(a)** AD = 11.3 cm, PQ = 2.67 cm **(b)** area of $ABCDE$ = 84.3 cm^2, area of $MNOPQ$ = 12.3 cm^2
34. **(a)** AB = 15.1 cm, $\angle BAD$ = 72.7° **(b)** 44.7 cm^2 **(c)** 20.5 cm^2
35. **(a)** no **(b)** 70.3°
36. **(a)** 3.57 cm **(b)** 25.4 cm^2 **(c)** 2.55 cm
37. **(a)** $\frac{1}{2}ar$ cm^2 **(c) (i)** 238 cm^2 **(ii)** 6.10 cm **(iii)** 23.8 cm